SEASONAL FLUCTUATIONS AND EQUILIBRIUM MODELS OF EXCHANGE RATE

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Abstract

Most of the evidence on dynamic equilibrium exchange rate models is based on seasonally adjusted consumption data. Equilibrium models have not worked well in explaining the actual exchange rate, but with seasonally adjusted data, there are reasons to expect spurious rejections of the model. This paper models exchange rate dynamics by means of an equilibrium model that incorporates seasonal preferences. The fit of the model to the data is evaluated for five industrialized countries using seasonally unadjusted data. Our findings indicate that a model with seasonal preferences can generate monthly time series of the exchange rate without seasonality, even if the variables that theoretically determine the exchange rate show seasonality. We also compare the stochastic properties of the theoretical exchange rate and observed exchange rate using cointegration analysis, finding we cointegration between both time series in some cases.

JEL classification: F31, F37, G15

Keywords: exchange rate, equilibrium model, seasonality.

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Abstract

Most of the evidence on dynamic equilibrium exchange rate models is based on seasonally adjusted consumption data. Equilibrium models have not worked well in explaining the actual exchange rate, but with seasonally adjusted data, there are reasons to expect spurious rejections of the model. This paper models exchange rate dynamics by means of an equilibrium model that incorporates seasonal preferences. The fit of the model to the data is evaluated for five industrialized countries using seasonally unadjusted data. Our findings indicate that a model with seasonal preferences can generate monthly time series of the exchange rate without seasonality, even if the variables that theoretically determine the exchange rate show seasonality. We also compare the stochastic properties of the theoretical exchange rate and observed exchange rate using cointegration analysis, finding we cointegration between both time series in some cases.

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1.- Introduction

Equilibrium models state that the exchange rate is determined by macroeconomic fundamentals such as money supplies, prices, outputs, and interest rates. In practice, however, fundamental variables have not proven helpful in predicting future changes in the exchange rate.

Engel and West (2003) do not find much evidence that the exchange rate is explained only by “observable” fundamentals and their view is that observable fundamentals do not explain most exchange rate changes. Thus, after nearly two decades of research following Meese and Rogoff’s pioneering work on exchange rate predictability (see Meese and Rogoff, 1983 a, b), the goal of exploiting equilibrium models of exchange rate determination to beat naïve random walk forecasts remains as elusive as ever (see Taylor, 1995). The question is why equilibrium models have not been very helpful in explaining changes in the exchange rate. Certain assumptions made in order to obtain a linear (easy-to-estimate) version of the equilibrium conditions of the theoretical exchange rate may be too restrictive. Moreover, equilibrium models state that the exchange rate is determined by macroeconomic fundamentals. Seasonality is inherent in such variables, but this statistical property does not appear in the exchange rate. Econometricians, rather than explicitly investigating the underlying economic seasonal variation, typically remove its effects by using seasonally adjusted data. Wallis (1974) shows that seasonal adjustment may distort the relations between variables.

These arguments lead us to analyze two possible explanations for the results obtained when an equilibrium model is used to explain exchange rate evolution: either (1) standard equilibrium models of exchange rate determination are inadequate to explain exchange rate evolution, or (2) the theory is fundamentally sound, but its empirical implementation is flawed. To shed light on this issue, Jimenez and Flores (2004) evaluate three different versions of equilibrium models of exchange rate determination, namely those of: i) Lucas (1982), ii) Svensson (1985), and (iii) Grilli and Roubini (1992). They show that standard versions of equilibrium models of exchange rate determination are unable to generate monthly time series with similar properties to
those observed in the actual exchange rate (high volatility and persistence and without autocorrelation at the seasonal lag).

Given these results in conjunction with those mentioned above, the key question for us is: Should something be changed in the model specifications in order for the equilibrium model to be able to replicate exchange rate properties (high volatility and persistence, without autocorrelation at the seasonal lag)?

In this article, we offer an economic framework that provides the answer to this question. We present a two-country open economy model that is a variation of the one developed in Grilli and Roubini (1992). Our model incorporates preferences with “taste shocks”, as in Miron (1986). The key idea underlying our strategy is based on the concept that nominal exchange rates are asset prices. Obstfeld and Rogoff (1996, p. 529) state, “One very important and quite robust insight is that the nominal exchange rate must be viewed as an asset price…”. Ferson and Harvey (1992) examined consumption-based asset pricing models using seasonally unadjusted consumption data. These authors suggest that in an asset pricing model using seasonally unadjusted data, there is a need to control the strong seasonality in consumption expenditure that is not reflected in asset returns. To do so, they incorporate seasonal taste shifts in preferences and estimate these together with other model parameters.

When seasonal preferences are considered, we find that the theoretical exchange rate is a function not only of the share of money used for asset transaction, goods endowments and the return on equities, as in previous equilibrium models, but also of seasonal shifts in preferences. The rationale underlying this is that consumers know that fundamental variables present seasonal variations, but their welfare improves if they smooth them over the entire the year, thus avoiding the creation of intertemporal distortions when making their decision to invest. This may explain an exchange rate without seasonality.

We subsequently evaluate the fit of our model to the data in four steps: (i) we obtain the GMM estimation of the structural parameters of the model using seasonally unadjusted data; (ii) we generate theoretical monthly time series of the exchange rate using the model equilibrium conditions evaluated at the point estimates of structural parameters using the corresponding money, production and asset returns observed; (iii)
we analyze stochastic properties of theoretical exchange rates using Box-Jenkins methodology; and finally, (iv) we compare the stochastic properties of the theoretical and observed exchange rate using cointegration analysis.

The results indicate that our model, which considers preferences with seasonal effects, overcomes the problem to evaluate equilibrium models of exchange rate induced by the seasonal fluctuation in fundamental variables. Even if seasonality appear in the variables that theoretically explain the exchange rate, our model could generate time series for some exchange rates with similar properties to the actual rate, i.e. (i) no seasonal fluctuations, and (ii) a degree of integration equal to 1. Furthermore, the same order of integration between actual and simulated exchange rate time series leads to the study of cointegration. This property has been found in some cases, but the cointegration vector only presented the right sign (i.e. the model replicates the observed depreciation of the British pound) for the British pound / US dollar exchange rate.

The paper is organized as follows. Section 2 presents a two-country, two-goods cash-in-advance (CIA) model of the exchange rate; in this section we describe a specific utility function with seasonal shocks. We evaluate our model in Section 3. To begin with, the parameters are estimated applying the GMM estimator over stochastic Euler equations, we then generate time series of the theoretical exchange rate for several currencies and finally compare the stochastic properties of the theoretical and observed exchange rate. Concluding remarks appear in Section 6. Appendix 1 contains a description of the data. Appendix 2 show the diagnostic analysis of GMM estimation. Appendix 3 presents time series data on observed and theoretical exchange rates.

2.- The economy

This section presents a CIA two-country economy that is subject to seasonal variation in preferences. A predecessor of this model is the work of Grilli and Roubini (1992), which analyzed the open economy implications of models in which money is used both for transactions on goods markets and for transactions on asset markets. In this paper, however, in contrast to Grilli and Roubini, securities are traded instead of bonds and preferences with seasonal shocks are specified. First, we describe the
economy and analyze the main implications of our model specifications for the equilibrium exchange rate. We then present the preferences, which are particularly important for the outcome of our analysis. Finally, the theoretical equilibrium exchange rate is obtained as an explicit function on fundamental variables and preference parameters.

A.- The two-country exchange rate model

There are two countries: Domestic (D) and Foreign (F). Each country has a firm, each produces a perishable, traded, distinct good: \( Y_t^D \), \( Y_t^F \). Firms are assumed to be able to sell claims of their future random outputs. Domestic (Foreign) claims entitle the owner to a proportionate share in the future stream of dividends.

Each country has the same number of private agents. There is one representative agent in each country. The representative agent from countries \( i, \) for \( i = D, F, \) has preferences described by infinite-horizon expected utility functions given by:

\[
E_t \left[ \sum_{s=t}^{\infty} \beta^{s-t} U \left( c_{it}^D, c_{it}^F \right) \right] \quad 0 < \beta < 1,
\]

where \( E_t \) denotes the mathematical conditional expectation on information known at the beginning of period \( t, \) \( \beta \) is a constant discount factor. Function \( U \) is assumed to be bounded, continuously differentiable, increasing in both arguments, and strictly concave. \( c_{ij}^j, \) for \( j = D, F, \) is the good produced in country \( j \) that is consumed by a resident of country \( i. \) Note that preferences are assumed to be identical across countries.

Residents of country \( i \) are allowed to own shares in productions in either country as well as the currency. The pattern of trading is assumed to proceed in a similar way to Lucas (1990). The transaction technology is that of the cash-in-advance model,

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1 We assume the convenient artifact of a three-member representative household, each of whom goes his own way during a period, the three reuniting at the end of a day to pool goods, assets, and information. One member of the household (the owner of the firm) collects the productions, which he must then sell to other households on a cash-in-advance basis. A household cannot consume any of its own endowments. Cash received from a sale on date-\( t \) cannot be used for any purpose during period \( t. \) A second member of the household takes an amount \( M_t - Z_t \) of the household’s initial cash balances \( (M_t) \) and uses it to purchase goods from other households. The
extended by the assumption that the representative agent faces two liquidity constraints: on the purchase of goods and on the purchase of assets. Domestic (foreign) goods may be bought only with domestic (foreign) currency. Agents are thus subject to the following cash-in-advance constraints. Let $N_i^j$ be the amount of money of country $j$ held by the representative agent of country $i$ for transactions in the goods market at time $t$, and let $P_i^j$ be the domestic and foreign currency prices for good $j$. Then we require:

$$N_i^D \geq P_i^D c_i^D, \quad i = D, F,$$

(2)

$$N_i^F \geq P_i^F c_i^F, \quad i = D, F,$$

(3)

Constraint (2) pertains to the currency of the domestic country, whereas (3) pertains to the currency of the foreign country. We now analyze equilibrium price determination. Equilibrium conditions for the currency for transactions on the goods markets are:

$$N_i^j = N_{iD}^j + N_{iF}^j, \quad j = D, F$$

(4)

Substituting (2)-(3) at equality in (4) gives:

$$N_i^j = P_i^j \left(c_i^{jD} + c_i^{jF}\right), \quad j = D, F.$$  

(5)

Equilibrium in the goods markets requires:

$$Y_i^j = c_i^{jD} + c_i^{jF}, \quad j = D, F.$$  

(6)

Substituting (6) in (5) gives the quantitative theory equations:

$$P_i^D = N_i^D / Y_i^D$$

(7)

$$P_i^F = N_i^F / Y_i^F$$

Therefore, the equilibrium prices of the two goods depend on domestic and foreign money supply.

stock agents start out with the money that was paid to them as dividends by domestic and foreign firms during this period. The third member carries out the remaining domestic and foreign currency cash balances on the securities market where domestic and foreign securities are sold and bought, and where he can buy and sell domestic and foreign currencies on the foreign exchange market. In addition, domestic (foreign) assets can be bought only with domestic (foreign) cash balances. Only two securities are assumed to exist in the securities market: domestic and foreign equity claims (shares in domestic and foreign future outputs).
In addition, there are two types of equities in this economy: domestic and foreign. These equities are traded on the securities market at a price (in units of domestic (foreign) currency) $Q^D_t \ (Q^F_t)$. During the securities trading session, the agent has access to the foreign exchange market and can choose holdings of domestic and foreign securities. He therefore faces the budget constraints given by:

$$[M^D_{it} - N^D_{it}] + S[M^F_{it} - N^F_{it}] = Q^D_t \omega^D_{it} + S_t Q^F_t \omega^F_{it}, \ i = D, F,$$

where $S_t$ is the nominal spot exchange rates expressed as the domestic price for foreign currency, $M^j_{it}$ are holdings of money $j$ on date $t$, and $\omega^j_{it}$ is the number of equities of country $j$ purchased at $t$ by a resident of country $i$.

In each period, the firm from country $j$ issues an amount of equities, the value of which has to be equal to its production. Thus:

$$P^j_t Y^j_t = (\omega^D_{it} + \omega^F_{it}) Q^j_t \ j = D, F,$$  

(9)

At the beginning of period $t+1$, the ownership of equities entitles the owner to receive the dividend and to have the right to sell the equity at price $Q^j_{t+1}$. Dividends are random because they depend on production, which is assumed to be governed by an unspecified stochastic process. Therefore, the agent will begin $t+1$ with cash balances given by

$$M^D_{it+1} = d^D_{it+1} \omega^D_{it} + Q^D_{it+1} \omega^D_{it} \ i = D, F$$  

(10)

$$M^F_{it+1} = d^F_{it+1} \omega^F_{it} + Q^F_{it+1} \omega^F_{it} \ i = D, F,$$  

(11)

where $d^j_t$ are dividends per equity of firm $j$ ($j=\text{D, F}$).

The domestic agent’s optimization problem may now be represented as follows. The agent chooses stochastic processes for $\{N^D_{it}, N^F_{it}, \omega^D_{it}, \omega^F_{it}\}$ to maximize (1) subject to the cash-in-advance constraints (2)-(3), the budget constraint (8) and the transition equation for state variables (10)-(11). The agent’s decision problem motivates Bellman’s equation,

$$V(M^D_{it}, M^F_{it}) = \text{Max } U\left(N^D_{it} / P^D_t, N^F_{it} / P^F_t \right) + \beta E_t \left[V\left(M^D_{it+1}, M^F_{it+1}\right)\right]$$

(12)

Note that our ultimate goal is the simulation of exchange rate time series. The simulation methodology does not involve positing a statistical model for the exogenous forcing variables.
First order and envelope conditions associated with the problem stated in (12) are used to characterize equilibrium behavior, assuming that the value functions exist and that they are increasing, differentiable and concave:

\[
\frac{\partial V}{\partial N^D_t} = U_{c_D} \left( c^D_{c_D}, c^F_{c_D} \right) \left( P^D_t \right)^{-1} = \lambda_t \tag{13}
\]

\[
\frac{\partial V}{\partial N^F_t} = U_{c_F} \left( c^D_{c_F}, c^F_{c_F} \right) \left( P^F_t \right)^{-1} = \lambda_t S_t \tag{14}
\]

\[
\frac{\partial V}{\partial \omega^D_{Dt}} = Q^D_t \lambda_t = \beta E_t \left[ V_{M^D_{t+1}} \left( d^D_{t+1} + Q^D_{t+1} \right) \right] \tag{15}
\]

\[
\frac{\partial V}{\partial \omega^F_{Dt}} = Q^F_t \lambda_t S_t = \beta E_t \left[ V_{M^F_{t+1}} \left( d^F_{t+1} + Q^F_{t+1} \right) \right] \tag{16}
\]

where \( \lambda_t \) is the multiplier associated with the budget constraint.

The envelope conditions are:

\[
V^*_{M^D_{t+1}} = U_{c_D} \left( c^D_{c_D}, c^F_{c_D} \right) \left( P^D_{t+1} \right)^{-1} \tag{17}
\]

\[
V^*_{M^F_{t+1}} = U_{c_F} \left( c^D_{c_F}, c^F_{c_F} \right) \left( P^F_{t+1} \right)^{-1} \tag{18}
\]

Substituting (13) and (17) in (15), the domestic firm equity price is given by:

\[
Q^D_t = \beta E_t \left( \frac{U_{c_D} \left( c^D_{c_D}, c^F_{c_D} \right) \left( P^D_t \right)^{-1}}{U_{c_D} \left( c^D_{c_D}, c^F_{c_D} \right) \left( P^D_{t+1} \right)^{-1}} \left( d^D_{t+1} + Q^D_{t+1} \right) \right) \tag{19}
\]

Symmetrically, substituting (14) and (18) in (16), the foreign firm equity price is given by:

\[
Q^F_t = \beta E_t \left( \frac{U_{c_F} \left( c^D_{c_F}, c^F_{c_F} \right) \left( P^F_t \right)^{-1}}{U_{c_F} \left( c^D_{c_F}, c^F_{c_F} \right) \left( P^F_{t+1} \right)^{-1}} \left( d^F_{t+1} + Q^F_{t+1} \right) \right) \tag{20}
\]

Finally, the solution for the equilibrium exchange rate is obtained from first order conditions (15)-(16), and envelope conditions (17)-(18):

\[
S_t = \frac{U_{c_D} \left( c^D_{c_D}, c^F_{c_D} \right) \left( P^D_t \right)^{-1} \left( d^D_{t+1} + Q^D_{t+1} \right) \left( Q^D_t \right)^{-1}}{U_{c_F} \left( c^D_{c_F}, c^F_{c_F} \right) \left( P^F_t \right)^{-1} \left( d^F_{t+1} + Q^F_{t+1} \right) \left( Q^F_t \right)^{-1}} \tag{21}
\]
To generate time series of theoretical exchange rate, it is necessary to characterize the agent preferences; in the next section, we present our assumption on these preferences.

**B- Seasonal shifts in preferences**

The issue of interest is that although the exchange rate is not seasonal, seasonal movements are a feature of many economic data series that theoretically determine the exchange rate, such Gross Domestic Product (GDP), Industrial Product Index (IPI), consumption, monetary aggregates, etc. The evidence suggests that optimizing agents know that fundamental variables present seasonal variations and they smooth these when making an investment decision.

The question is: How can our equilibrium model explain this fact? One possible way is based on assuming preferences that include seasonal shocks. In equilibrium models, Euler equations imply asset pricing expressions that suggest tight links between asset prices and underlying investor preferences. It is natural that the price of an asset, in equilibrium, is the expected discounted value of its future payoff, weighted by a marginal utility of consumption (Lucas 1978). In our model, one of the first order conditions describing the investor’s optimal consumption and portfolio plan is:

$$
E_t \left( \frac{U_{c_{t+1}}(c^{D}_{t+1}, c^{F}_{t+1})}{U_{c_{t}}(c^{D}_{t}, c^{F}_{t})} R_{t+1} \right) = 1, \text{ for } j = D, F,
$$

(22)

where $R_{t+1}$ is one plus a real return on shares of country $i$. The function

$$
m_{t+1} = \beta U_{c_{t+1}}(c^{D}_{t+1}, c^{F}_{t+1}) \left(U_{c_{t}}(c^{D}_{t}, c^{F}_{t})\right)^{-1}
$$

is the measure of the representative agent’s intertemporal marginal rate of substitution (the discounted ratio of marginal utilities in two successive periods) on the good $j$. Note that $m_{t+1}$ is a function of consumption, and consumption usually possesses seasonality. Equation (22) implies that the seasonal pattern in consumption should be reflected in $R_{t+1}$ (however, the seasonal variation on asset returns, $R_{t+1}$, is minute compared with seasonal fluctuations in consumption) unless the agents remove seasonal fluctuations in
consumption (for example, through function $U_{ij}$) to a level that can be exactly mirrored in the real returns of all assets.

Therefore, we assume that the representative agents from both countries have identical preferences described by (Miron, 1986)\(^3\):

$$U\left(c^D_i(s), c^E_i(s)\right) = \frac{1}{1-\gamma^D} \left(c^D_i(s)\right)^{1-\gamma^D} + \frac{1}{1-\gamma^F} \left(c^E_i(s)\right)^{1-\gamma^F},$$

(23)

where $c^j_i(s)$ is the consumption service flow received in period $t$ by the agent from country $i$ for the consumption of the good produced in country $j$, in season $s$, and $1/\gamma^j$ is the intertemporal elasticity of substitution in consumption of the consumption service of the good produced in country $j$. Consumers transform the stock of the consumption good produced in country $j$, $c^j_i$, into consumption services according to the following function:

$$c^j_i(s) = \exp(\lambda^j_i) c^j_i, \quad j = D, F$$

(24)

$$\lambda^j_i = \theta^j_i \mu_i(1) + \theta^j_i \mu_i(2) + \ldots + \theta^j_i \mu_i(12)$$

where $\mu_i(s)$ is a dummy variable taking the value one when period $t$ corresponds to season $s$, and zero otherwise, and $\theta^j_i$ is the seasonal preference in season $s$. These parameters indicate that the utility obtained from $c^j_i(s)$ varies according to the prevailing season $s$. $c^j_i(s)$ may thus be thought of as the “seasonally adjusted” consumption.

\(^3\) We assume that the utility function is additively separable from the two consumption services and presents constant intertemporal substitution elasticity. Although this assumption is restrictive, it simplifies estimation considerably. Our analysis focuses on a simple basic utility function with seasonal shocks. We think this is valuable for setting a benchmark. Even so, the literature on equilibrium models of exchange rates includes heterogeneity of agents, multiple sector, tax shocks, and modifications designed to reproduce features of exchange rates. Whether our findings are relevant for these cases is an open, quantitative issue that may be addressed using the procedures implemented in this paper.
C.- Characterizing the equilibrium exchange rate

In order to describe an equilibrium solution of the model, we require a distribution of wealth. We have assumed that wealth of every kind is evenly divided between the residents of the two countries. Specifically, the residents of each country own half of the domestic and foreign endowments, the stocks of domestic and foreign money and domestic and foreign assets. Given this distribution of wealth, consumption is given by:

\[ c^D_i = \frac{Y^D_i}{2}, \quad i = D, F \]

\[ c^F_i = \frac{Y^F_i}{2}, \quad i = D, F \]

In addition, considering equilibrium prices given by (7) and the preferences defined above, (23)-(24), the equilibrium exchange rate given by expression (21) becomes:

\[
S_i = \frac{(1/2)^{y^F} E_t \left( \exp \left[ \sum_{s=1}^{12} \theta^F_{s} \mu_i(s) \right] Y^F_{i+1} \right)^{1/y^F} \left( N^F_{i+1} \right)^{-1} \left( d^F_{i+1} + Q^F_{i} \right) (Q^F_{i})^{-1}}{(1/2)^{y^D} E_t \left( \exp \left[ \sum_{s=1}^{12} \theta^D_{s} \mu_i(s) \right] Y^D_{i+1} \right)^{1/y^D} \left( N^D_{i+1} \right)^{-1} \left( d^D_{i+1} + Q^D_{i} \right) (Q^D_{i})^{-1}}
\]

Expression (26) suggests that the spot exchange rate is a function not only of money, goods endowments and the rate of return on equities between dates \( t \) and \( t+1 \), but is also a function of the seasonal shifts in preferences. This expression enables us to study how agents smooth the seasonality in fundamental variables through shifts in preferences.

Our model thus explains that, although seasonal fluctuation is a common property in economic activity \( \left( Y^j_i, N^j_i, j = D, F \right) \), the exchange rate is not seasonal. We argue that agents improve their welfare by smoothing the seasonality in fundamental variables, thus avoiding the creation of intertemporal distortions when making their decision to invest. In our model, this process is represented by the shifts

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4 This solution is the perfectly pooled equilibrium of Lucas (1982).
that appear in preferences. By lowering service consumption flows received at a time when goods endowments are high and increasing them at a time when goods endowments are low, a policy that smooths seasonal fluctuations in service consumption flows will increase welfare.

We can consider the economic model to be an approximation to the stochastic processes generating the actual data. If the economic model is correctly specified, it could generate data with similar characteristics to those of the observed data. Expression (26) may be used to compute the exchange rate time series implied by this model and then compare it with real data. The next section describes the way to do this.

3. Estimation of parameters and empirical evaluation of the seasonal model

In this section, we evaluate our model analyzing its capacity to generate time series of exchange rates with similar properties to observed exchange rates. We will use a procedure that consists of four steps:

1. First, we estimate the preference parameters of the model exploiting orthogonality conditions from our model using Hansen’s (1982) generalized method of moments (GMM). Theoretical variables are approximated by their observed counterparts. Outputs are approximated by monthly industrial production indexes and monetary aggregates are approximated by M2. Asset returns are generated by taking a first difference on the natural logarithm of the equity price index.

2. We then use expression (26) to generate theoretical exchange rate time series. The preference parameters are evaluated at their point estimates and the theoretical variables \( Y_t, M_t \), and assets returns are approximated by their observed counterparts.

3. We analyse the stochastic properties of theoretical exchange rates using Box-Jenkins methodology (Box and Jenkins, 1990).

4. Finally, we compare the stochastic properties of theoretical and observed exchange rates using cointegration analysis.

A. Details of the procedure for estimating preference parameters
In this section, we briefly describe the estimation procedure. We focus on GMM estimation as implemented for dynamic rational expectations models by Hansen and Singleton (1982). The dynamic optimization problem for an economic agent implies a set of stochastic Euler equations that must be satisfied in equilibrium. These Euler equations in turn imply a set of population orthogonality conditions that depend non-linearly on variables observed by econometricians and on unknown parameters characterizing preferences. These conditions of orthogonality are used to construct a criterion function whose minimizer is the estimate of parameters.

Though there are many different types of Euler equations for the model specified in the previous section, the parameters are estimated by exploiting expression (19), which may also be written as:

\[
\beta E \left[ \frac{U_{C}(c^D_{D,t+1}(s), c^F_{D,t+1}(s))}{U_{C}(c^D_{D,t}(s), c^F_{D,t}(s))} \left( \frac{P^I_{i,t+1}}{P^I_{i,t}} \right)^{-1} \left( d^I_{i,t+1} + Q^I_{i,t+1} \right) \left( Q^I_{i,t} \right)^{-1} \left| X_t \right. \right] = 1, \quad (27)
\]

where \( X_t \) is a vector representing all the information available to the agent at date \( t \).

Estimation using (27) requires the function \( U \) to be explicitly parameterized.

Therefore, when (23) is substituted in (27):

\[
E \left[ \beta \left\{ \exp \left( \sum_{k=2}^{12} \theta_k \left[ \mu_{\nu,k}(s) - \mu_{k}(s) \right] \right) \right\}^{1-p} \left( \frac{c^D_{D,t+1}}{c^D_{D,t}} \right)^{-p} \left( \frac{P^I_{i,t+1}}{P^I_{i,t}} \right) \left( d^I_{i,t+1} + Q^I_{i,t+1} \right) \left( Q^I_{i,t} \right)^{-1} \left| X_t \right. \right] = 0. \quad (28)
\]

The nominal price of good \( j \), for \( j=D, F \), is given by cash-in-advance constraint (7) and assuming pooling equilibria as in (25), expression (28) becomes:

\[
E \left[ \beta \left\{ \exp \left( \sum_{k=2}^{12} \theta_k \left[ \mu_{\nu,k}(s) - \mu_{k}(s) \right] \right) \right\}^{1-p} \left( \frac{Y^I_{i,t+1}}{Y^I_{i,t}} \right)^{-p} \left( \frac{N^I_{i,t+1}}{N^I_{i,t}} \right) \left( d^I_{i,t+1} + Q^I_{i,t+1} \right) \left( Q^I_{i,t} \right)^{-1} \left| X_t \right. \right] = 0. \quad (29)
\]

Expression (29) requires the random variable described by:

---

5 The first order conditions derived must hold with respect to all assets. The derivations, however, are carried out only for a single asset for each country.
\[
\beta \left\{ \exp \left( \sum_{s=2}^{12} \theta_s \left[ \mu_{s+1}(s) - \mu_s(s) \right] \right) \right\}^{1/\gamma} \left( \frac{Y_{t+1}'}{Y_t'} \right)^{-\gamma} \left( \frac{N_{t+1}'}{N_{t+1}} \right) \left( \frac{d_{t+1}'+Q_{t+1}'}{Q_t'} \right) - 1, \tag{30}
\]
to be uncorrelated with any variable contained in the information set \( X_t \). It should therefore be the case that:

\[
E \left( \beta \left\{ \exp \left( \sum_{s=2}^{12} \theta_s \left[ \mu_{s+1}(s) - \mu_s(s) \right] \right) \right\}^{1/\gamma} \left( \frac{Y_{t+1}'}{Y_t'} \right)^{-\gamma} \left( \frac{N_{t+1}'}{N_{t+1}} \right) \left( \frac{d_{t+1}'+Q_{t+1}'}{Q_t'} \right) - 1 \right) z_t = 0, \tag{31}
\]
where \( z_t \) is any subset of the information set \( X_t \) that we are able to observe.

Let \( \psi_{o'} = \{ \beta, \gamma, \theta_d, \theta_f, \theta_d', \theta_f', \theta_d', \theta_f', \theta_d', \theta_f', \theta_d', \theta_f', \theta_d', \theta_f', \theta_d', \theta_f', \theta_d', \theta_f' \} \subset \mathbb{R}^{14} \) denote the vector of unknown parameters, for \( j = D, F \), that are to be estimated, and let \( w_{t+1} = \left( Y_{t+1}', Y_t', N_{t+1}', \left( \frac{N_{t+1}'}{N_{t+1}} \right), \left( d_{t+1}'+Q_{t+1}'/Q_t' \right), \left( \mu_{s+1}(s) - \mu_s(s) \right) \) \) denote the vector of variables that are observed by agents for date \( t+1 \). Hence, we can define the function \( h(w_{t+1}, \psi_{o'}) \) given by:

\[
h(w_{t+1}, \psi_{o'}) = \beta \left\{ \exp \left( \sum_{s=2}^{12} \theta_s \left[ \mu_{s+1}(s) - \mu_s(s) \right] \right) \right\}^{1/\gamma} \left( \frac{Y_{t+1}'}{Y_t'} \right)^{-\gamma} \left( \frac{N_{t+1}'}{N_{t+1}} \right) \left( \frac{d_{t+1}'+Q_{t+1}'}{Q_t'} \right) - 1. \tag{32}
\]

We can interpret \( (32) \left( u_{t+1} = h(w_{t+1}, \psi_{o'}) \right) \) as the disturbance vector in our econometric estimation, which should have finite second moments and, given (31),

\[
E \left( u_{t+1} \right| X_t \) = 0. \tag{33}
\]

We can now define the criterion function \( f \) given by:

\[
f(w_{t+1}, z_t, \psi_{o'}) = h(w_{t+1}, \psi_{o'}) \otimes z_t, \tag{34}
\]
where \( z_t \) is the \( q \) dimensional vector of variables with finite second moments that are in the agent’s information set; and \( f \) maps \( \mathbb{R}^q \times \mathbb{R}^{14} \) into \( \mathbb{R}^q \) and \( \otimes \) is the Kronecker product. Thus, an implication of (33)-(34) and their accompanying assumptions is that

---

\(^{6}\) The utility function assumed in (23) is additively separable over time and additively separable in domestic and foreign goods. This assumption allows us to estimate seasonal taste parameters, factor discount and intertemporal elasticity of substitution parameters using single equation methods for each good. This can be seen by noting that in (32), \( u_{t+1} \) is a function only of the variables \( Y_t, M_t \) and asset returns corresponding to country \( j (=D, F) \).
Equation (35) represents a set of \( q \) population orthogonality conditions from which an estimator of \( \psi_0 \) may be obtained, provided that \( q \) is at least as large as the number of unknown parameters.

We proceed by constructing an objective function that depends only on available sample information and the unknown parameters. Let

\[
g(\psi') = E\left[f\left(w_{t+1}, z_t, \psi' \right)\right],
\]

where \( \psi' \in \mathbb{R}^{14} \). Note that (35) implies that \( g(\psi') \) has a zero at \( \psi' = \psi_0' \). Thus, the GMM estimator of \( g(\psi') \)

\[
g_T(\psi') = \frac{1}{T} \sum_{t=1}^{T} f\left(w_{t+1}, z_t, \psi' \right),
\]

evaluated at \( \psi' = \psi_0' \), \( g_T(\psi_0') \), should be close to zero for large values of \( T \). Given this fact, it is reasonable to select the \( \psi' \) that makes \( g_T(\psi') \) “close” to zero. Therefore, a GMM estimator of \( \psi_0' \) can be obtained by minimizing the quadratic form

\[
J_T [\psi'] = \left[ g_T (\psi') \right] W_T \left[ g_T (\psi') \right],
\]

where \( W_T \) is a \( q \) by \( q \) symmetric positive definite matrix that can depend on sample information. An important aspect of specifying a GMM problem is the choice of the weighting matrix. We use the optimal \( W_T = \hat{\Omega}^{-1} \), where \( \hat{\Omega} \) is the estimated covariance matrix of the sample moments \( q \). We use consistent Two-Stage least square estimates for the initial estimate of \( \psi_0' \) in forming the estimate of \( \Omega \).\(^8\)

**Estimation**

To estimate \( \psi_0' \), we use monthly seasonally unadjusted data from 1986:01 to 1998:04 for five countries: Germany (GM), Spain (SP), Japan (JP), the United Kingdom (UK), and the United States (US). Outputs are approximated by the corresponding

\[\begin{align*}
&1, Y_{t}/Y_{t-1}, \ldots, Y_{t-11}/Y_{t-12}, M_{t}/M_{t-1}, \ldots, M_{t-11}/M_{t-12}, \log (Q_{t}/Q_{t-1}), \ldots, \\
&\log (Q_{t-11}/Q_{t-12})
\end{align*}\]

\(^7\) The instruments are: a constant term, lagged production growth rates, lagged monetary aggregates growth rates, and lagged rates of return: \( z_t = \{1, Y_{t}/Y_{t-1}, \ldots, Y_{t-11}/Y_{t-12}, M_{t}/M_{t-1}, \ldots, M_{t-11}/M_{t-12}, \log (Q_{t}/Q_{t-1}), \ldots, \\
\log (Q_{t-11}/Q_{t-12})\} \)

\(^8\) We use Heteroskedasticity and an Autocorrelation Consistent Covariance Matrix of the sample moments.
monthly industrial production indexes (IPI), monetary aggregates by the corresponding M2, and asset returns are generated by taking a first difference on the natural logarithm of the equity price index. Appendix 1 describes the data and their stochastic properties.9

Table 1 shows the GMM estimation of $\psi^j_s$. The seasonal taste coefficients $\theta^j_s$, for $s=1, 2, 3, \ldots, 11$, and $j=GM, SP, JP, UK, US$ indicate how the model seasonally adjusts consumption levels in month $s$ relative to December for the good produced in country $i$. December is chosen as a reference month ($\theta^j_{12}=1$). Seasonal taste parameters are statistically significant in most cases. The fact that seasonal preference shocks are significantly different from zero means that these shocks must be included in order to explain the joint behavior of consumption and asset returns.

The estimates of $\gamma^j$ are similar to those found in other studies, ranging from 0.74 to 2.42.10 In order to test the validity of overidentifying restrictions, Hansen’s (1982) $J$-statistics are also displayed. The null hypothesis (overidentifying restrictions are satisfied) is not rejected at a 5% significance level in any case.

9 Hansen (1982) showed that sufficient conditions for the asymptotic properties of the GMM include strict stationarity of the data. Strict stationarity may be violated for some kinds of seasonal variation. However, consistency and asymptotic normality of the estimators and the asymptotic distribution of the test statistic can be demonstrated under weaker conditions. See Jagannathan (1983) and Lim (1985) for an analysis of the asymptotic properties of the GMM under seasonality and non-stationarity. Stationarity may also be violated under some models of growth rates of real outputs or monetary aggregates. In our empirical study, these growth rates are stationary.

10 In their study of aggregate fluctuations, Kydland and Prescott (1982) found that they needed a value of between one and two to mimic the observed relative variability of consumption and investment. Altug (1983) estimates the parameter at near zero. Kehoe (1983), studying the response of small countries’ balance of trade to terms of trade shocks, obtains estimates near one. Hildreth and Knowles (1986), in their study of the behavior of farmers, also obtain estimates between one and two. Mehra and Prescott (1985) present evidence for restricting the value of relative risk aversion to a maximum of ten, though without specifying a concrete value. Eichenbaum et. al. (1984), Mankiw et al. (1985) and Hansen and Singleton (1982) report values of $\gamma$ between zero and one. Mankiw (1985) reports values of between 2 and 4.
Table 1: GMM estimation of utility function parameters  

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
<th>$\theta_5$</th>
<th>$\theta_6$</th>
<th>$\theta_7$</th>
<th>$\theta_8$</th>
<th>$\theta_9$</th>
<th>$\theta_{10}$</th>
<th>$\theta_{11}$</th>
<th>$J_{Stat}$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1^{GM}$ (b)</td>
<td>0.130* (0.06)</td>
<td>0.121** (0.07)</td>
<td>0.112 (0.08)</td>
<td>0.125 (0.09)</td>
<td>0.079 (0.08)</td>
<td>0.156** (0.08)</td>
<td>0.270* (0.09)</td>
<td>0.066 (0.12)</td>
<td>-0.061* -0.022*</td>
<td>1.31* (0.18)</td>
<td>29.52</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_1^{SP}$</td>
<td>-0.115* (0.03)</td>
<td>-0.153* -0.255* -0.262* -0.349* -0.328* -0.264* 0.344* -0.106* -0.133* -0.139* 0.74*</td>
<td>0.033 (0.08)</td>
<td>0.112 (0.09)</td>
<td>0.125 (0.09)</td>
<td>0.079 (0.09)</td>
<td>0.156** (0.08)</td>
<td>0.270* (0.10)</td>
<td>-0.061* -0.022*</td>
<td>1.31* (0.18)</td>
<td>29.52</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_1^{JP}$</td>
<td>0.139* (0.03)</td>
<td>0.107* -0.038* 0.090</td>
<td>0.154* 0.019</td>
<td>0.032</td>
<td>0.137* 0.012</td>
<td>0.003 -0.005</td>
<td>1.51* (0.04)</td>
<td>18.45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_1^{UK}$</td>
<td>0.047 (0.05)</td>
<td>0.012 -0.182* -0.063</td>
<td>-0.112 -0.151 -0.003</td>
<td>0.096 -0.053 -0.118* -0.106*</td>
<td>1.22* (0.05)</td>
<td>20.97</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_1^{US}$</td>
<td>0.022* (0.004)</td>
<td>0.001 -0.007* -0.002</td>
<td>0.017* -0.054* 0.001 -0.031* -0.042* -0.037* -0.014*</td>
<td>2.42* (0.004)</td>
<td>25.83</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
(a) The instruments are: a constant term, lagged production growth rates, lagged monetary aggregates growth rates, and lagged rates of return.
(b) Germany (GM), Spain (SP), Japan (JP), United Kingdom (UK), United States (US)
(c) Estimated standard errors in brackets
(d) Statistical significance is indicated by a single asterisk (*) for the 5% level, and a double asterisk (**) for the 10% level
(e) J-statistic, for testing the validity of overidentifying restrictions. Under the null hypothesis, the overidentifying restrictions are satisfied, the J-statistic (i.e. the minimized value of the objective function) times the number of observations is asymptotically $\chi^2_q$, with degrees of freedom equal to the number of overidentifying restrictions
(f) P values represented in brackets.

Figures 1-5, in Appendix 2, show the residual graphs, autocorrelation function (ACF) and partial autocorrelation function (PACF) associated with GMM equations. Residuals are white noise.

B- Stochastic properties of theoretical and observed exchange rates

We restrict our testing strategy to the model implications in the statistical properties of the exchange rate. Once the entire parameters vector $\psi_0$ is estimated, using the corresponding IPI, M2 and the returns on assets, we generate several theoretical monthly time series implied by expression (26), evaluated at the point estimates of the utility function parameters of Table 1: German mark (DEM/USD) 11, Japanese yen (JPY/USD), Spanish peseta (ESP/USD), and British pound (GBP/USD) relative to the US dollar, as well as Japanese yen (JPY/DEM), Spanish peseta (ESP/DEM), and British pound (GBP/DEM) relative to the German mark.

11 The currency is calculated as the value of the second country’s currency. For example, (GBP/USD) is the number of British pounds needed to purchase a US dollar; in this case the UK is the domestic country and the USA the foreign country.
Table 2 reports a variety of descriptive statistics of the theoretical exchange rate, \((ThtExRa)\) and the observed exchange rate \((ObsExRa)\) over the sample period 1990:01-1998:04.\(^\text{12}\) Mean (M), standard deviation (Std), skewness (Skw), kurtosis (Kt) and the order of integration (d). The stochastic process of the \(ThtExRa\) time series is then analyzed and compared with the stochastic processes characterizing the \(ObsExRa\). Table 2 shows time series analysis results; diagnostic checks are developed to detect model inadequacy. Descriptive statistics of the residuals from estimated models are reported: mean \((\mu)\) and estimated mean standard error \((\hat{\sigma}_\mu)\), estimated standard errors \((\hat{\sigma})\) and Ljung-Box Q-statistics at lag 12 to test for serial correlation \((Q(12))\).

### Table 2: Summary of ARIMA\(^\text{13}\) models fitted to the \(ThtExRa\) and the \(ObsExRa\)

<table>
<thead>
<tr>
<th></th>
<th>M</th>
<th>Std.</th>
<th>Skw</th>
<th>Kt</th>
<th>(\hat{\sigma}_\mu)</th>
<th>(\hat{\sigma})</th>
<th>Q(12)</th>
<th>ARIMA MODELS(^\text{13}(\text{b}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs</td>
<td>DEM/USD</td>
<td>1.6*10^6</td>
<td>1.2*10^-3</td>
<td>0.03</td>
<td>4.6*10^2</td>
<td>9.7</td>
<td>0.20 (\xi_{3947} + N_t)</td>
<td>(\nabla N_t = a_t)</td>
</tr>
<tr>
<td>Tht</td>
<td>DEM/USD</td>
<td>2.1*10^6</td>
<td>1.9*10^-3</td>
<td>0.07</td>
<td>4.7*10^2</td>
<td>12.6</td>
<td>(1-0.16B+0.32B^2) (V Y_t = a_t)</td>
<td></td>
</tr>
<tr>
<td>Obs</td>
<td>ESP/USD</td>
<td>1.2*10^6</td>
<td>1.6*10^0</td>
<td>0.07</td>
<td>3.4*10^1</td>
<td>0.3*10^0</td>
<td>0.9</td>
<td>(Y_t = (7.5 + 10.2 B) \xi_{3947} + 12.7 \xi_{40} + N_t)</td>
</tr>
<tr>
<td>Tht</td>
<td>ESP/USD</td>
<td>6.8*10^4</td>
<td>8.1*10^-5</td>
<td>0.07</td>
<td>2.6*10^2</td>
<td>6.6</td>
<td>(1-0.52B+0.15B^2+0.23B^7) (V Y_t = a_t)</td>
<td></td>
</tr>
<tr>
<td>Obs</td>
<td>GBP/USD</td>
<td>6.1*10^6</td>
<td>4.6*10^2</td>
<td>0.07</td>
<td>1.5*10^1</td>
<td>6.6</td>
<td>(7.5 + 10.2 B) (\xi_{3947} + N_t)</td>
<td>(\nabla N_t = a_t)</td>
</tr>
<tr>
<td>Tht</td>
<td>GBP/USD</td>
<td>4.6*10^6</td>
<td>4.4*10^2</td>
<td>0.07</td>
<td>7.3*10^2</td>
<td>10.9</td>
<td>(1-0.37B+0.25B^2+0.18B^4) (V Y_t = a_t)</td>
<td></td>
</tr>
<tr>
<td>Obs</td>
<td>JPY/USD</td>
<td>6.1*10^6</td>
<td>1.0*10^-1</td>
<td>0.16</td>
<td>3.4*10^1</td>
<td>0.3*10^0</td>
<td>0.9</td>
<td>(Y_t = (0.08 +0.08B) \xi_{3947} + N_t)</td>
</tr>
<tr>
<td>Tht</td>
<td>JPY/USD</td>
<td>4.6*10^6</td>
<td>4.4*10^2</td>
<td>0.07</td>
<td>7.3*10^2</td>
<td>10.9</td>
<td>(1-0.37B+0.25B^2+0.18B^4) (V Y_t = a_t)</td>
<td></td>
</tr>
<tr>
<td>Obs</td>
<td>GBP/DEM</td>
<td>3.8*10^6</td>
<td>4.0*10^-1</td>
<td>0.22</td>
<td>1.3*10^2</td>
<td>6.5</td>
<td>(0.08 +0.08B) (\xi_{3947} + N_t)</td>
<td>(\nabla N_t = a_t)</td>
</tr>
<tr>
<td>Tht</td>
<td>GBP/DEM</td>
<td>2.2*10^6</td>
<td>6.6*10^-1</td>
<td>0.10</td>
<td>3.6*10^2</td>
<td>10</td>
<td>(1-0.21B+0.41B^2) (V Y_t = a_t)</td>
<td></td>
</tr>
<tr>
<td>Obs</td>
<td>JPY/DEM</td>
<td>7.4*10^6</td>
<td>0.9*10^-1</td>
<td>0.47</td>
<td>2.4*10^2</td>
<td>8.5</td>
<td>(Y_t = a_t)</td>
<td></td>
</tr>
<tr>
<td>Tht</td>
<td>JPY/DEM</td>
<td>9.7*10^-1</td>
<td>1.0*10^-3</td>
<td>0.33</td>
<td>3.1*10^2</td>
<td>17.3</td>
<td>(1-0.34B+0.23B^2) (V Y_t = a_t)</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
(a) Estimated standard errors in brackets

\(^\text{12}\) Preference parameters were estimated with the whole sample; nonetheless, we analyze this shorter period in order to compare with Jimenez and Flores’ previous results.

\(^\text{13}\) Autoregressive Integrated Moving Average (ARIMA) Model
The stochastic processes for all currencies (observed and theoretical) are modeled in first differences. Outliers in ObsExRa are analyzed to conclude that the random walk process is a valid representation for six out of the seven analyzed cases. The first difference in the GBP/DEM time series behaves as a first order autoregressive process.

ThtExRa is generated by expression (26) using seasonally unadjusted data (IPI and M2); seasonality in these variables is controlled in the model. The time series analysis shows that autocorrelation of ThrExRa at the seasonal lag does not exist, as in the autocorrelation of ObsExRa. Figures 6-12 in Appendix 3 plot monthly data from 1990:01 to 1998:04 for both theoretical and observed exchange rates.

C.- Cointegration analysis

The fourth column in Table 2 shows that ObsExRa and ThtExRa time series follow integrated processes of order 1 [I(1)]; given this result we can test for cointegration. We use the simplest procedure, sometimes called the Engle-Granger or EG test (Engle and Granger, 1987), which involves first using OLS to estimate the following cointegration regression:

\[
\text{ObsExRa}_t = \beta_0 + \beta_1 \text{ThtExRa}_t + u_t^{\text{ExRa}},
\]

and then using an ordinary Dickey Fuller test based on the regression:

\[
\nabla u_t^{\text{ExRa}} = \mu + (1 - \alpha) u_{t-1}^{\text{ExRa}} + \epsilon_t.
\]

Since serial correlation may be a problem, it is more common to use an Augmented Engle-Granger or AEG test, which is related to the EG test in the same way as the ADF test is related to the ordinary DF test. If \(u_t^{\text{ExRa}}\) is I(0), regression (38) implies that the variables ObsExRa and ThtExRa will be cointegrated with the cointegrating vector \((1, -\beta_1)\).

We also perform a cointegration test using the Trace (Tr_ST) and Maximum eigenvalue (MaxAu_ST) tests developed by Johansen (1991, 1992). Table 3 shows the results.
Within this framework, cointegration means that the economic model replicates the long-run evolution of the actual exchange rate. Moreover, if the OLS estimate of \(\beta_1\) in (38) is positive, the economic model subsequently also replicates the appreciation or depreciation process in the observed time series.

Table 3. Testing for cointegration between ObsExRa and ThtExRa.

<table>
<thead>
<tr>
<th>ExRa</th>
<th>AEG</th>
<th>Tr_ST(c)</th>
<th>MaxAu_ST</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(b_0^{\beta})</td>
<td>(\beta_1)</td>
<td>(\tau^0)</td>
</tr>
<tr>
<td>DEM/USD</td>
<td>0.2<em>10^{-1} (0.1</em>10^{-1})</td>
<td>-1.5<em>10^{-3} (0.6</em>10^{-3})</td>
<td>-2.54</td>
</tr>
<tr>
<td>ESP/USD</td>
<td>17<em>10^{-3} (0.8</em>10^{-3})</td>
<td>-9.3<em>10^{-3} (1.1</em>10^{-3})</td>
<td>-2.32</td>
</tr>
<tr>
<td>GBP/USD</td>
<td>2.0<em>10^{-3} (0.9</em>10^{-4})</td>
<td>6.6<em>10^{-3} (0.8</em>10^{-3})</td>
<td>-3.26**</td>
</tr>
<tr>
<td>JPY/USD</td>
<td>8.4<em>10^{-1} (1.0</em>10^{-3})</td>
<td>1.6<em>10^{-3} (0.5</em>10^{-3})</td>
<td>-1.21</td>
</tr>
<tr>
<td>ESP/DEM</td>
<td>2.0<em>10^{-2} (0.2</em>10^{-2})</td>
<td>-3.9<em>10^{-3} (0.5</em>10^{-3})</td>
<td>-3.23**</td>
</tr>
<tr>
<td>GBP/DEM</td>
<td>8.4<em>10^{-3} (1.2</em>10^{-4})</td>
<td>-0.2<em>10^{-4} (0.6</em>10^{-4})</td>
<td>-1.79</td>
</tr>
<tr>
<td>JPY/DEM</td>
<td>1.3<em>10^{-3} (0.6</em>10^{-4})</td>
<td>6.3<em>10^{-3} (0.6</em>10^{-3})</td>
<td>-2.80</td>
</tr>
</tbody>
</table>

Notes:
(a) Estimated standard errors in brackets
(b) \(\tau^0\) are computed in exactly the same way as the ordinary t statistic for \(\alpha I=0\) in regression (39). The lag length (Lag) of the lagged difference terms of the dependent variable on the right hand side of (39) is determined using the Akaike information criterion (AIC). Maximum number of lag = 12. Critical values for the AEG test are taken from Davidson and MacKinnon (1993). –3.90 (1%), -3.34 (5%), -3.04 (10%). Statistical significance is indicated by a single asterisk (*) for the 5% level, and a double asterisk (**) for the 10% level.
(c) The system variables are (ObsExRa and ThtExRa). The lag value indicates the order of the vector error correction model (VECM) estimated for each currency, which is determined using the Akaike information criterion (AIC). The asymptotic critical values (without a constant in the data generating process, although the cointegrating equations have intercepts) obtained from Osterwald-Lenum (1992) are presented in the following table, in which \(p\) is the number of system variables and \(h\) is the number of cointegration relations under the null hypothesis. Trz are the critical values for Johansen’s likelihood ratio test of the null hypothesis of \(h\) cointegration relations against the alternative of NO restrictions. Max are the critical values for Johansen’s likelihood ratio test of the null hypothesis of \(h\) cointegration relations against the alternative of \(h+1\) relations. Statistical significance is indicated by a single asterisk (*) for the 5% level, and a double asterisk (**) for the 10% level.

Test results are mixed:
1.- At the 10% critical value, the \(\tau^c\) statistic in Table 3 suggests that estimated residuals \(\hat{u}_t^{GBP/USD}\) are I(0). Johansen tests also provide evidence of cointegration among the ObsExRa and ThtExRa time series at the 10% critical value. Additionally, as \(\beta_1\) is positive for this currency in regression (38), the economic model also replicates the
appreciation or depreciation observed in the actual time series. Figure 7 in Appendix 3 shows the ObsExRa and ThtExRa time series.

2.- The AEG and Johansen tests reported in Table 3 for the ESP/DEM exchange rate suggests, at the 10% significance level, that observed and theoretical time series appear to be cointegrated, but \( \hat{\beta} < 0 \); i.e. the economic model forecasts an appreciation when real data shows depreciation. The ESP/DEM exchange rate shows outliers in 10/92 and 5/93 due to the European Monetary System crisis. However, an intervention analysis reveals that cointegration tests were not distorted.

3.- For the remaining currencies, the null hypothesis of no cointegration is accepted at the 10% significance level. In these cases, although the economic model is able to remove seasonality from the exchange rate, it is not able to replicate other important features related to long-run evolution.

5. Conclusions

Perhaps the most important characteristic of equilibrium models of the exchange rate is that they enable the discussion of evidence on the predictability of exchange rates from fundamental variables. Exchange rate equilibrium models illustrate how fundamental variables may affect the dynamics of the exchange rate. These models generate equilibrium pricing functions relating the exchange rate to real production, monetary aggregates and asset returns. However, there is little evidence that the exchange rate is explained by fundamental variables that theoretically determine it.

This paper generalizes standard dynamic equilibrium models by allowing for seasonal shocks in preferences. In contrast with prior studies, the theoretical model is tested directly with seasonal unadjusted data. We found that the model with seasonal shocks in preferences explains how agents smooth seasonal movements in fundamental variables when they make their decisions to invest. Our model is able to reproduce the stochastic process of the actual exchange rate for some currencies: (1) no seasonal fluctuation and (2) degree of integration equal to one. For instance, in the case of the GBP/USD rate, the model captures the long-term patterns and the depreciation found in the data.
We view the result of this paper as providing a certain counterbalance to the bleak view in relation to the usefulness of the equilibrium model of exchange rate determination. We have focused on a simple, basic equilibrium exchange rate model, but we believe this to be a valuable contribution as regards setting a benchmark.
References


Appendix 1.- The data

Monthly seasonally unadjusted data from 1986:01 to 1998:04 are used for five countries: Germany (GM), Spain (SP), Japan (JP) United Kingdom (UK), and United States (US). The monetary aggregate, M2, is taken from EcoWin. The Industrial Production Index (IPI) is used as a proxy for income, and is compiled from the OCDE. The exchange rates of the German mark (DEM), Japanese yen (JPY), Spanish peseta (ESP), and British pound (GBP) relative to the US dollar are taken from the OCDE.

Asset return data are generated by taking the first difference of the natural logarithm of the equity price index: DAX-XETRA (DAX) for GM, the General Index of the Madrid Stock Exchange (IGBM) is sufficiently representative of the Spanish stock exchange, the Nikkei-225 index (NIKKEI) is used for JP, the FT-100 (FT) for the UK, and Dow-Jones (DJ) for the US, (December 1994=100). Stock index data are taken from the Financial Times, London. Table 4 below reports time series analyses. Prior to the simulation, we start out by checking for the presence of extreme values. We performed intervention analysis [Box and Tiao, 1975]. Time series analysis of the data indicates that these series do not display mean-reversion and hence they are a I(1) process. IPI and M2 series show very regular seasonal patterns. The random walk process is consistent with the data generating process of the exchange rate and the stock index. All stock indexes show extreme values in the 1987 October crash.
Appendix 2: Diagnostic analysis of GMM estimation: residual graphs, ACF and PACF.

Figure 1
GERMANY

Figure 2
SPAIN

Figure 3
JAPAN
Appendix 3: *ObsExRa* and *ThtExRa*

Figure 6

*Obs DEM / USD (Left) & Tht DEM / USD (Right)*

Figure 7

*Obs GBP / USD (Left) & Tht GBP / USD (Right)*

Figure 8

*Obs ESP / USD (Left) & Tht ESP / USD (Right)*
Figure 9

Obs JPY / USD (Left) & Th t JPY / USD (Right)

Figure 10

Obs GBP / DEM (Left) & Th t GBP / DEM (Right)

Figure 11

Obs ESP / DEM (Left) & Th t ESP / DEM (Right)
Figure 12

*Obs JPY / DEM (Left) & Tht JPY / DEM (Right)*