SMALL SAMPLE POWER OF BARTLETT CORRECTED LIKELIHOOD RATIO TEST OF COINTEGRATION RANK

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Abstract

It is well-documented phenomenon that the asymptotic distribution of the likelihood ratio test of cointegration rank is different from the small sample distribution. In the aftermath, size and power of the test are substantially distorted and the null hypothesis is rejected too often when it is true. One of the standard solutions of such problem is a correction of test statistic, so that the small sample distributions and its cumulants were closer to the asymptotic counterparts. The Bartlett correction for the trace test derived by Johansen for the multivariate case constitutes a convenient tool for plausible inference on cointegration rank. However, the decline of empirical size of the test entails the obvious decrease of power. The paper investigates the power of the Bartlett corrected trace test with reference to the power of the usually applied uncorrected one and tries to answer for which hypotheses, sample sizes and parameter’s spaces it is satisfactory.

1. INTRODUCTION

The vector autoregressive framework and the superconsistent maximum likelihood estimator has became a popular method of modelling of vector stochastic processes integrated of order one (see Engle and Granger (1987), Johansen (1988)). However, a sore point of the cointegration analysis is the inference on the cointegration rank, especially if the samples are short and the data not very informative. Amid all tests used to discriminate between different hypotheses on the cointegration rank, the trace test is mostly used and in some cases, examined by Lütkepohl (2001), has better small sample properties when a number of cointegrating relations exist, as compared with competitive maximum likelihood test.
The limit distribution of the trace test statistic is given as the trace of stochastic matrix, which depends on multidimensional Brownian motion and deterministic components present in the cointegration space. It is approximated by means of simulations for long sample sizes (see Johansen (1988, 1991), Ahn and Reinsel (1990)). It should be noted that the trace test is consistent, and it follows that asymptotically the size of the test for the true null hypothesis is maintained if the “top – down” strategy of inference is applied (see Juselius (2003)). However, it turned out that the size and the power of the test are severely distorted in small samples when the statistic is juxtaposed with the asymptotic critical values (see Toda (1995), cf. Haug (1996) for maximum eigenvalue test and others). The distortions result from the fact, that small sample distribution of statistic and its expected value depends not only on the number of common stochastic trends and the deterministic component but also on the number of observations and the true parameters of the vector autoregressive model. Therefore, a number of solutions were proposed and employed to reduce the deviation of the expected value of the trace test statistics in small sample from the expected value in the limit. They can be in general divided into two groups, one trying to estimate the actual critical value for given sample size and another one, trying to bring closer the expected value of the statistic to the asymptotic critical value by some correction of the statistic. To the former group fall the response surfaces (see Cheung and Lai (1993), MacKinnon et al. (1998)) as well as the bootstrap and the Edgeworth expansions (see Rothenberg (1984), Giersbergen (1996)), whereas to the latter one fall Bartlett and Bartlett-Type corrections (see Bartlett (1937), Cribari-Neto and Cordeiro (1996)) and other correction factors (see Reimers (1992)).

In the paper we shall focus on the Bartlett correction for the trace test, and the emphasis will be laid on small sample properties of corrected trace test when the alternative hypothesis is true. The motivation results from the fact, that the uncorrected trace test suffers from the lack of power in short samples (see Toda (1995)), whereas the Bartlett correction for the trace test has to result in a decrease of power, as the empirical size of the test is reduced in most cases (although the correction factor can take also fractional value as well as unsatisfactorily the negative one). Thus, for plausible inference it seems to be crucial to know, for which sample sizes, which hypotheses, and which values of parameters the Bartlett corrected trace test is a powerful tool.

The idea of the Bartlett correction is quite intuitive and amounts to bringing closer the expected value of statistic in finite sample to expected value of statistic in the limit, with hope that the other moments of the distribution are also better approximated. Therefore, suppose that the likelihood ratio statistic converges to its asymptotic critical value with an error of order at most $T^{-1}$ (and it is fulfilled indeed), i.e.:

$$E[-2\ln(LR)] = E[\lim_{T \to \infty}(-2\ln(LR))] + O(T^{-1}),$$

(1)
then the Bartlett correction relies on the existence of such expansion that
\[
E[-2\ln(LR)] = E[\lim_{T \to \infty}(-2\ln(LR))] \left(1 + \frac{b}{T} + O(T^{-2})\right),
\]  
where constant \( b \) can be consistently estimated under the null hypothesis, and on the convergence of \(-2\ln(LR)/E[-2\ln(LR)]\) to \(\lim_{T \to \infty}(-2\ln(LR))/E[\lim_{T \to \infty}(-2\ln(LR))]\), as \( T \to \infty \) (see Bartlett (1937)). Hence, the Bartlett correction can be derived from the approximate equality:
\[
\lim_{T \to \infty}(-2\ln(LR)) \approx -2\ln(LR) \cdot \frac{E[\lim_{T \to \infty}(-2\ln(LR))]}{E[-2\ln(LR)]}.
\]

Nevertheless, the assumption that whole shape of distribution is better approximated by Bartlett correction has to be verified. In fact, unlike the classical inference when regularity conditions holds, the LR statistics is \( \chi^2 \) distributed, and all quantiles are approximated with an error of order \( T^{-3/2} \) (see Lawley (1956)), in vector autoregressive framework with unit roots the improvement is not so spectacular, however for some cases a better fit exist (see Bravo (1999)).

In the next section, we will recall the Bartlett correction for the trace test and do some remarks on its small sample properties. Section 3 include the design of several Monte Carlo experiments, the results concerning the power of the Bartlett corrected trace test and some hints for econometricians willing to apply the corrected test. Section 4 gives the summary.

2. THE BARTLETT CORRECTION FOR THE TRACE TEST

The Bartlett correction for the trace test statistic in bivariate autoregressive framework has been worked out by Jacobson and Larsson (1999), while the formula for the general multivariate autoregressive model with unit roots was derived by Johansen (2002), who suggested the approximation of the expected value of the statistic under the null hypothesis which takes the form:
\[
E[-2\ln(LR)] = f(T, n_b, n_d) \left(1 + \frac{b(\hat{\theta})}{T}\right),
\]
where \( n_b \) denotes number of common stochastic trends, \( n_d \) defines the maximum power of linear trend restricted to the cointegration space and \( b(\hat{\theta}) \) expresses the aforementioned constant depending on estimated parameters of the model under the null. Since \( E[\lim_{T \to \infty}(-2\ln(LR))] = f(n_b, n_d) \), the correction factor is given as:
\[
a(T, n_b, n_d)\left(1 + \frac{b(\hat{\theta})}{T}\right),
\]
where \( a(T, n_s, n_d) = f(T, n_s, n_d)/f(n_s, n_d) \). Johansen proposes to calculate (5) by means of simulation and presents the estimated coefficients of the approximation, which are closely related to response surfaces regression. The term \( b(\hat{\theta}) \) is derived analytically and after neglecting one insignificant component is given as:

\[
b(\hat{\theta}) = c_1(1 + h(n_s, n_d)) + (n_s c_2 + 2(c_1 + n_s c_1)) g(n_s, n_d) / n_s^2,
\]

where \( h(n_s, n_d) \) and \( g(n_s, n_d) \) are again calculated by means of simulations, while \( c_1, c_2, c_3 \) are the coefficients, which are calculated on the basis of the companion form parameters of the vector autoregressive model under the null, which in turn is as follows:

\[
\Delta X_t = a(\beta'X_{t-1} + \rho't^n) + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \sum_{i=0}^{n-1} \Phi_i t^i + \epsilon_t,
\]

and the companion form of (7) is:

\[
Y_t = PY_{t-1} + Q\epsilon_t.
\]

After omitting the deterministic terms, as insignificant for the formula given by equation (6) (see Johansen (2002)), the companion form can be written as:

\[
\begin{bmatrix}
\beta'X_t \\
\Delta X_t \\
\Delta X_{t-1} \\
\vdots \\
\Delta X_{t-k+2}
\end{bmatrix} = \begin{bmatrix}
I_r + \beta'\alpha & \beta'T_1 & \cdots & \beta'T_{k-2} & \beta'T_{k-1} \\
\alpha & \Gamma_1 & \cdots & \Gamma_{k-2} & \Gamma_{k-1} \\
0 & I_n & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & I_n & 0
\end{bmatrix} \begin{bmatrix}
\beta'X_{t-1} \\
\Delta X_{t-1} \\
\Delta X_{t-2} \\
\vdots \\
\Delta X_{t-k+1}
\end{bmatrix} + \begin{bmatrix}
\beta' \\
I_r \\
0 \\
\vdots \\
0
\end{bmatrix} \epsilon_t,
\]

where \( n \) denotes number of stochastic variables and \( k \) defines number of lags in vector autoregressive model.

It should be highlighted, that the Bartlett correction changes the small sample properties of the trace test in other manner than the corrections derived by Ahn and Reinsel as well as by Reimers do. As presented by Johansen, size of the uncorrected trace test grows towards one, as the stochastic process becomes almost integrated in second order, while size of the Bartlett corrected trace test tends to zero in such circumstances. This is due to the fact that the Bartlett correction handles the distortion which result from both small sample bias and dependence on the parameters under the null, whereas the above-mentioned corrections handle only the former cause. Such property of the Bartlett corrected trace test has to pose the question, what is the power of the test in small samples, and how the short term dynamics as well as other parameters of the model under the null hypothesis influence the power.
To the best knowledge of the author, no evidence on power of the Bartlett corrected trace test is available, apart from two figures calculated by Johansen (2002), which relate to an empirical model.

3. POWER OF THE BARTLETT CORRECTED TRACE TEST

The analysis of power of the Bartlett corrected trace test was conducted in three distinct sections. First of all, the power was calculated for two different sample sizes: $T = 50$ and $T = 100$ due to the dependence of the distribution of the statistics on the sample size (see Johansen (1996) and (2002)) and in accordance with the assumed sample sizes in the latter of the aforementioned papers, in which small sample properties of the Bartlett corrected trace test are considered with reference to the probability of type I error. Secondly, the power was analysed for different ‘distances’ between the assumed null hypothesis and the true alternative because of the fact, that the more non-zero eigenvalues is present in the statistic the easier the null should be rejected. Finally, the power for different number of cointegration relations under the null hypothesis was considered, in order to investigate, whether the test has the same power in the small sample when the alternative, that one more cointegration relation exists, is true irrespective of number of cointegration relations in the null hypothesis, assuming that the number of common trends is constant.

The Monte Carlo technique was employed for the calculations. For each experiment 100 thousand replications were computed. The vector autoregressive model under the alternative and the assumed data generating process is as follows (cf. Johansen (2002)):

$$
\Delta X_t = a\left(\beta'X_{t-1} + \rho'\epsilon_{t-1}\right) + a\left(\beta'X_{t-1} + \rho'\epsilon_{t-1}\right) + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \sum_{i=0}^{\eta_i-1} \Phi_i't + \epsilon_t.
$$

In all simulations, the only one deterministic component was constant restricted to the cointegration space, i.e. $n_d = 0$, number of lags for short term structure was assumed as $k = 2$ and variance-covariance matrix was $\Omega = I_n$.

3.1. The power for distinct sample sizes

As mentioned above, the distribution of the trace test statistics depends on the sample size and the Bartlett correction reduces the size of trace test in general. Therefore it is of crucial importance to know, for which sample sizes the Bartlett corrected trace test is powerful tool in comparison to the uncorrected test. However, it should be stressed, that the latter often leads to false conclusions, as the size of the test is seriously distorted.
The power was calculated for the set of $n = 5$ stochastic variables, the DGP was given by the equation (9), true cointegration rank was $r^{DGP} = 2$, whereas tested null hypothesis was $r = 1$ versus at the most $r = 5$ under the alternative (notice that for the sake of the way the asymptotic critical values are calculated, the hypotheses are reverted in comparison with the likelihood ratio statistic, for which the null hypothesis have to ensure the maximum value of the statistic). The parameters of DGP were given as follows:

$$\alpha = [0 0 0 0 0]^\top, \quad \beta = [1 0 -1 0 0]^\top, \quad \rho = [1],$$

$$\alpha_i = [0 0 0 0 0]^\top, \quad \beta_i = [0 1 0 -1 0]^\top, \quad \rho_i = [1], \quad \Gamma_i = \xi I_5.$$  

At the outset, sample size $T = 50$ was assumed, next $T = 100$. Tables 1 and 2 present the results for different values of loadings and autoregressive coefficients.

<table>
<thead>
<tr>
<th>$\alpha \setminus \xi$</th>
<th>0</th>
<th>0.1</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1</td>
<td>9.5 (1.24)</td>
<td>11.8 (1.03)</td>
<td>19.7 (1.31)</td>
<td>37.4 (1.38)</td>
<td>72.3 (1.54)</td>
<td>99.6 (2.12)</td>
</tr>
<tr>
<td>-0.2</td>
<td>14.7 (1.21)</td>
<td>17.9 (1.22)</td>
<td>28.5 (1.26)</td>
<td>48.6 (1.33)</td>
<td>82.4 (1.48)</td>
<td>99.8 (2.34)</td>
</tr>
<tr>
<td>-0.4</td>
<td>28.5 (1.18)</td>
<td>33.5 (1.19)</td>
<td>48.1 (1.22)</td>
<td>71.0 (1.29)</td>
<td>95.2 (1.44)</td>
<td>99.9 (2.20)</td>
</tr>
<tr>
<td>-0.6</td>
<td>42.4 (1.15)</td>
<td>48.5 (1.17)</td>
<td>65.4 (1.20)</td>
<td>87.0 (1.27)</td>
<td>99.1 (1.43)</td>
<td>99.9 (2.14)</td>
</tr>
<tr>
<td>-0.8</td>
<td>54.1 (1.14)</td>
<td>60.9 (1.15)</td>
<td>78.5 (1.19)</td>
<td>95.2 (1.26)</td>
<td>99.9 (1.63)</td>
<td>99.9 (2.10)</td>
</tr>
<tr>
<td>-0.9</td>
<td>59.0 (1.14)</td>
<td>66.4 (1.15)</td>
<td>83.8 (1.19)</td>
<td>97.1 (1.26)</td>
<td>99.9 (1.42)</td>
<td>99.9 (2.19)</td>
</tr>
</tbody>
</table>

The figures in fraction present the simulated power of the uncorrected (above) and Bartlett corrected (below) trace test respectively, for nominal type I error equal 0.05. The Bartlett correction factor is in parenthesis.

<table>
<thead>
<tr>
<th>$\alpha \setminus \xi$</th>
<th>0</th>
<th>0.1</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1</td>
<td>8.7 (1.13)</td>
<td>10.8 (1.14)</td>
<td>17.8 (1.16)</td>
<td>32.0 (1.19)</td>
<td>64.3 (1.26)</td>
<td>99.5 (1.64)</td>
</tr>
<tr>
<td>-0.2</td>
<td>26.3 (1.11)</td>
<td>31.5 (1.11)</td>
<td>46.6 (1.13)</td>
<td>68.6 (1.16)</td>
<td>94.6 (1.23)</td>
<td>99.9 (1.60)</td>
</tr>
<tr>
<td>-0.4</td>
<td>70.3 (1.08)</td>
<td>77.3 (1.09)</td>
<td>89.6 (1.10)</td>
<td>97.9 (1.14)</td>
<td>99.9 (1.22)</td>
<td>99.9 (1.60)</td>
</tr>
<tr>
<td>-0.6</td>
<td>92.7 (1.07)</td>
<td>95.5 (1.08)</td>
<td>98.9 (1.10)</td>
<td>99.9 (1.13)</td>
<td>99.9 (1.21)</td>
<td>99.9 (1.59)</td>
</tr>
<tr>
<td>-0.8</td>
<td>98.5 (1.07)</td>
<td>99.3 (1.07)</td>
<td>99.9 (1.09)</td>
<td>99.9 (1.13)</td>
<td>99.9 (1.21)</td>
<td>99.9 (1.58)</td>
</tr>
<tr>
<td>-0.9</td>
<td>99.3 (1.06)</td>
<td>99.7 (1.07)</td>
<td>99.9 (1.09)</td>
<td>99.9 (1.13)</td>
<td>99.9 (1.21)</td>
<td>99.9 (1.58)</td>
</tr>
</tbody>
</table>

The figures in fraction present the simulated power of the uncorrected (above) and Bartlett corrected (below) trace test respectively, for nominal type I error equal 0.05. The Bartlett correction factor is in parenthesis.
As it turned out, when testing the cointegration rank for $T=50$ (see Table 1), the power of the Bartlett corrected trace test is rather unsatisfactory for such short sample and if there can be indicated such a parameter space that the uncorrected trace test has sufficient low probability of type II error with possibly large probability of type I error (see Johansen (2002)), there is almost no space, for which the results for the corrected test could be considered as satisfactory. The increment of sample size to $T=100$ essentially changes the conclusions (see Table 2). The Bartlett corrected trace test is fairly powerful for moderate loadings of about $-0.5$ and the larger the autoregressive coefficients are, the higher the power of the test is. For $\alpha = -0.4$ and $\xi = 0.7$ we are able to indicate at true alternative hypothesis in 99%, whereas the probability of type I error is under control, as the statistic was Bartlett corrected.

The unsatisfactory results concerning the power of the Bartlett corrected trace test for $T=50$ put the question, whether the inference is more reliable when a ‘distance’ between verified null hypothesis and true alternative is larger than one non-zero eigenvalue, i.e. two or more cointegration relations should be still detected.

3.2. The power for different ‘distances’ between null and alternative hypotheses

The DGP is defined as in equation (9), the number of stochastic variables $n = 5$ and the true cointegration rank $r^{\text{DGP}} = 2$ remain unchanged. The null hypothesis is verified that $r = 0$ versus at the most $r = 5$ under the alternative hypothesis. The parameters of DGP are as follows:

$$\alpha = \theta, \quad \alpha_1 = \begin{bmatrix} \alpha & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \beta_1 = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \end{bmatrix}, \quad \rho_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad I = \xi I_5,$$

and the results for $T = 50$ are reported in Table 3.

<table>
<thead>
<tr>
<th>$\alpha \backslash \xi$</th>
<th>0</th>
<th>0.1</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1</td>
<td>38.9</td>
<td>44.2</td>
<td>58.6</td>
<td>78.2</td>
<td>96.0</td>
<td>99.9</td>
</tr>
<tr>
<td>-0.2</td>
<td>51.1</td>
<td>57.4</td>
<td>72.3</td>
<td>88.5</td>
<td>99.0</td>
<td>99.9</td>
</tr>
<tr>
<td>-0.4</td>
<td>75.2</td>
<td>80.9</td>
<td>91.3</td>
<td>98.4</td>
<td>99.9</td>
<td>99.9</td>
</tr>
<tr>
<td>-0.6</td>
<td>89.8</td>
<td>93.1</td>
<td>98.1</td>
<td>99.9</td>
<td>99.9</td>
<td>99.9</td>
</tr>
<tr>
<td>-0.8</td>
<td>96.0</td>
<td>97.7</td>
<td>99.7</td>
<td>99.9</td>
<td>99.9</td>
<td>99.9</td>
</tr>
<tr>
<td>-0.9</td>
<td>97.4</td>
<td>98.6</td>
<td>99.9</td>
<td>99.9</td>
<td>99.9</td>
<td>99.9</td>
</tr>
</tbody>
</table>

The figures in fraction present the simulated power of the uncorrected (above) and Bartlett corrected (below) trace test respectively, for nominal type I error equal 0.05. The Bartlett correction factor is in parenthesis.
As expected, an additional non-zero eigenvalue has caused an increase in power of the Bartlett corrected trace test as well as in power of the uncorrected one. Therefore, for high values of loadings, i.e. at least \(-0.6\) or even \(-0.8\), accompanied by non-zero autoregressive coefficients the power could be considered as sufficient. Therefore, taking into consideration the results presented in Tables 1 and 3, it is almost impossible to conduct plausible and accurate inference on the true cointegration rank by means of the Bartlett corrected trace test for short samples of about fifty observations, that is for such case that the probability of type I error is controlled by the Bartlett correction, the probability of type II error is sufficiently low and no information on the dimension of cointegration space is given \textit{a priori}. However, when the adjustments to the long-run attractors are fast and the stochastic variables are autocorrelated, the corrected test is capable to give some approximation.

It should be noted, that when the autoregressive coefficients are close to 1, and the process \(X_t\) becomes almost integrated of order second, then the power of the corrected test decline, because the size of the corrected test decrease towards 0 (see Johansen (2002)), as is revealed by the last column of Table 3 and a part of the last column of Table 1.

The next question which should be stated is whether, for given number of common trends and constant ‘distance’ between the null and alternative hypothesis, the probability of type II error rise, if the cointegration rank under the null does the same.

3.3 The power for different dimensions of vector stochastic processes

It is well known that the trace test is consistent as well as the Bartlett corrected one, as the correction factor is bounded. Thus, in the limit there is no difference in terms of power (and size too) when we test different null hypotheses, if the number of common trends is constant and the alternative hypothesis is true (the null for size respectively). Moreover, if the alternative hypothesis is true, then regardless of the number of common stochastic trends the power is the same and equal to 1. Nevertheless, in the small sample the expectation on power of the test cannot be the same. Therefore two additional matrices of results of Monte Carlo experiments were calculated, to be compared with the results presented in Table 1.

Firstly, the dimension of process \(X_t\) was set to \(n=4\), the DGP was defined as in equation (9), the true cointegration rank was \(r^{DGP}=1\), the null hypothesis was \(r=0\) and sample size \(T=50\), so that the number of common stochastic trends was the same as in the experiment reported in Table 1. The parameters of DGP were as follows:

\[
\begin{align*}
\alpha & = \theta, \\
\alpha_i &= [\alpha_0 \ 0 \ 0]^T, \\
\beta_i &= [1 \ 0 \ -1 \ 0]^T, \\
\rho_1 &= [1], \\
\Gamma_1 &= \xi I_4.
\end{align*}
\]

The results are presented in Table 4.
Table 4

<table>
<thead>
<tr>
<th>$\alpha \backslash \xi$</th>
<th>0</th>
<th>0.1</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1</td>
<td>22.7/8.8 (1.13)</td>
<td>25.4/8.6 (1.14)</td>
<td>34.1/8.6 (1.20)</td>
<td>49.2/7.8 (1.29)</td>
<td>75.6/4.6 (1.52)</td>
<td>99.2/0.2 (2.64)</td>
</tr>
<tr>
<td>-0.2</td>
<td>30.0/12.6 (1.13)</td>
<td>34.1/12.9 (1.14)</td>
<td>45.3/13.4 (1.20)</td>
<td>62.2/13.3 (1.29)</td>
<td>88.0/11.9 (1.52)</td>
<td>99.9/4.1 (2.64)</td>
</tr>
<tr>
<td>-0.4</td>
<td>47.3/23.6 (1.13)</td>
<td>52.3/24.6 (1.14)</td>
<td>65.5/27.3 (1.20)</td>
<td>83.7/31.7 (1.29)</td>
<td>98.3/43.3 (1.52)</td>
<td>99.9/26.9 (2.64)</td>
</tr>
<tr>
<td>-0.6</td>
<td>62.2/35.9 (1.13)</td>
<td>67.7/37.9 (1.14)</td>
<td>81.2/44.2 (1.20)</td>
<td>94.7/55.5 (1.29)</td>
<td>99.8/73.7 (1.52)</td>
<td>99.9/51.6 (2.64)</td>
</tr>
<tr>
<td>-0.8</td>
<td>73.6/48.0 (1.13)</td>
<td>78.8/50.9 (1.14)</td>
<td>90.4/60.0 (1.20)</td>
<td>98.4/75.8 (1.29)</td>
<td>99.9/89.8 (1.52)</td>
<td>99.9/69.6 (2.64)</td>
</tr>
<tr>
<td>-0.9</td>
<td>78.0/53.1 (1.13)</td>
<td>83.1/56.7 (1.14)</td>
<td>93.3/67.6 (1.20)</td>
<td>99.3/83.5 (1.29)</td>
<td>99.9/93.8 (1.52)</td>
<td>99.9/76.2 (2.64)</td>
</tr>
</tbody>
</table>

The figures in fraction present the simulated power of the uncorrected (above) and Bartlett corrected (below) trace test respectively, for nominal type I error equal 0.05. The Bartlett correction factor is in parenthesis.

Table 5

<table>
<thead>
<tr>
<th>$\alpha \backslash \xi$</th>
<th>0</th>
<th>0.1</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1</td>
<td>6.1/0.1 (1.28)</td>
<td>8.1/0.2 (1.32)</td>
<td>15.7/0.3 (1.42)</td>
<td>36.8/0.6 (1.51)</td>
<td>79.2/2.2 (1.65)</td>
<td>99.8/14.2 (2.53)</td>
</tr>
<tr>
<td>-0.2</td>
<td>9.0/0.3 (1.28)</td>
<td>11.9/0.4 (1.29)</td>
<td>22.6/0.5 (1.29)</td>
<td>43.7/1.3 (1.39)</td>
<td>82.8/5.6 (1.53)</td>
<td>99.9/15.3 (2.13)</td>
</tr>
<tr>
<td>-0.4</td>
<td>19.9/1.8 (1.22)</td>
<td>23.8/2.3 (1.23)</td>
<td>38.4/4.0 (1.26)</td>
<td>62.6/8.5 (1.31)</td>
<td>92.3/21.1 (1.47)</td>
<td>99.9/24.2 (2.09)</td>
</tr>
<tr>
<td>-0.6</td>
<td>30.1/3.5 (1.19)</td>
<td>36.9/6.7 (1.19)</td>
<td>53.2/11.0 (1.22)</td>
<td>79.0/22.7 (1.28)</td>
<td>97.5/42.3 (1.44)</td>
<td>99.9/35.5 (2.19)</td>
</tr>
<tr>
<td>-0.8</td>
<td>41.0/11.0 (1.16)</td>
<td>47.9/13.4 (1.18)</td>
<td>68.5/22.6 (1.21)</td>
<td>90.7/40.7 (1.27)</td>
<td>99.4/64.7 (1.44)</td>
<td>99.9/47.9 (2.26)</td>
</tr>
<tr>
<td>-0.9</td>
<td>45.3/13.8 (1.16)</td>
<td>53.7/17.9 (1.17)</td>
<td>73.6/29.5 (1.20)</td>
<td>93.7/50.7 (1.27)</td>
<td>99.7/73.2 (1.44)</td>
<td>99.9/53.3 (3.02)</td>
</tr>
</tbody>
</table>

The figures in fraction present the simulated power of the uncorrected (above) and Bartlett corrected (below) trace test respectively, for nominal type I error equal 0.05. The Bartlett correction factor is in parenthesis.

Secondly, the number of stochastic variables was set as $n = 6$, the DGP was defined as in equation (9), the true cointegration rank was $r_{DGP} = 3$, the null hypothesis was $r = 2$ and sample size $T = 50$, so again the number of common stochastic trends was the same. The parameters of DGP were given as:

\[
\alpha = \theta, \quad a_1 = \begin{bmatrix} \alpha & 0 & 0 & 0 & 0 & \end{bmatrix}^{\prime}, \quad \beta_1 = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \ 0 & 1 & 0 & 0 & -1 & 0 \ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix}, \quad \rho = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \Gamma_1 = \xi I_6,
\]

the results are in Table 5.

Comparison of Tables: 4, 1 and 5 respectively, reveals that for every values of parameters $\alpha$ and $\xi$ the power of the Bartlett corrected trace test diminishes, as the number of the cointegration relations under the null hypothesis increases for researched hypotheses. The result
does not allow to conclude that the power of the Bartlett corrected trace test declines uniformly and monotonically as the number of cointegration relations under the null hypothesis rises, however it enables to state, that in short samples it is better to build cointegrated systems with a limited number of stochastic variables and cointegration relations. The similar remarks can be also made for not reported here sample size $T = 100$.

4. CONCLUSIONS

The analysis of the power of the Bartlett corrected trace test was conducted in three directions. First, the dependence of the probability of type II error on the sample size was investigated and it was found that properties of the corrected test in terms of power are clearly better and satisfactory for the moderate sample sizes of about 100 observations, in contrast to the case of the short sample of about 50 observations, for which the inference is not plausible. Next, power of the corrected test was examined for the case, when there is more than one non-zero eigenvalue that should be detected as the alternative hypothesis is true, in comparison to verified null hypothesis. The result is that there is some parameter space, for which the probability of type II error is sufficiently small, even if there is only 50 observations available. Finally, power of the corrected test is analysed for different hypotheses on the cointegration rank under the null, whereas the number of common stochastic trends is constant and the true cointegration rank is higher by one than the null assumes. As it turned out, it is more likely to reject the false null hypothesis if the dimension of vector stochastic process is small.

An overall conclusion is that for short samples of about 50 observations the result of inference with the Bartlett corrected trace test should be considered as a proxy for the true cointegration rank and possibly additional \textit{a priori} information is needed. For moderate samples of about 100 observations, if residuals are multivariate white noise process, the inference is valid.
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