Volatility versus Correlation Risk in Dynamic Asset Allocation: A Bayesian Perspective

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Abstract

This paper assesses the relative economic value of volatility and correlation risk in the context of multivariate dynamic asset allocation strategies. Using exchange rate data, we model the dynamic covariance matrix of daily returns by implementing the multivariate Asymmetric Dynamic Conditional Correlation (ADCC) model of Cappiello, Engle and Sheppard (2006). Our statistical analysis develops a new Bayesian estimation algorithm for the ADCC model, provides a ranking of alternative model specifications in a way that accounts for parameter uncertainty, and constructs combined forecasts across a large set of correlation and volatility specifications using Bayesian Model Averaging. More importantly, we assess the economic value of volatility and correlation timing for the optimal portfolio decision of a risk averse investor in a dynamic mean-variance framework. We find that in foreign exchange markets there is substantial economic value in timing correlations in addition to the economic value of volatility timing; the former can add up to 350 basis points per annum to the 500 basis points of the latter. This result is robust to reasonably high transaction costs as well as alternative volatility specifications, diagonal correlation structure and asymmetric correlations.

Keywords: Asset Allocation; Dynamic Conditional Correlation; Correlation Timing; Volatility Timing; Bayesian MCMC Estimation; Bayesian Model Averaging.

JEL Classification: C11; C53; F31; F37; G11.

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1 Introduction

Active management of large portfolios requires dynamic forecasts of the expected returns, variances and covariances of asset returns. This has spurred a long line of research in financial econometrics exploring tractable multivariate volatility models. Prominent multivariate extensions of traditional univariate volatility models in the GARCH or stochastic volatility framework include Bollerslev, Engle and Wooldridge (1988), Diebold and Nerlove (1989), Harvey, Ruiz and Shephard (1994), Jacquier, Polson and Rossi (1994), Engle and Kroner (1995), Ledoit, Santa-Clara and Wolf (2003), Fiorentini, Sentana and Shephard (2004), Gourieroux, Jasiak and Sufana (2005), Chib, Nardari and Shephard (2006), Philipov and Glickman (2006a,b) and Asai and McAleer (2006). In this context, the Dynamic Conditional Correlation (DCC) model of Engle (2002) provides a parsimonious, flexible and easy to estimate multivariate framework for modeling the dynamics of volatilities and correlations of asset returns. Therefore, it has become a benchmark multivariate model for realistic applications of optimal asset allocation over a large number of assets.

Recent evidence suggests that correlation risk may be substantially more important than volatility risk in intertemporal portfolio problems (Buraschi, Porchia and Trojani, 2006). However, despite the growing econometrics literature using the DCC model, there is no study to date assessing the relative economic value of volatility and correlation timing using the DCC model for designing realistic dynamic asset allocation strategies, especially for the foreign exchange (FX) market. For example, Engle and Sheppard (2001) examine the return volatility of global minimum variance portfolios implied by the DCC model, but do not explicitly measure the economic cost of constant conditional correlations. Engle and Colacito (2006) introduce a set of formal statistical tests for comparing covariance estimators in the context of minimum volatility allocation strategies. However, their study is largely bivariate over short samples, ignores parameter uncertainty, and more importantly, it does not provide a utility evaluation of dynamically rebalanced portfolios from the perspective of a risk averse investor.

The DCC model has yet to be estimated and evaluated using a Bayesian methodology. An advantage of the Bayesian approach is that it provides the posterior distribution of the model parameters, which holds for finite samples. The posterior distribution can then be used to rank a set of alternative specifications in a way that accounts for both parameter and model uncertainty across univariate volatility and multivariate correlation specifications. A Bayesian approach also avoids the complication on testing the null of constant correlation against the alternative of dynamic conditional correlation that arises due to lack of identification of the correlation decay parameters (see Engle and Sheppard, 2001).

Our empirical investigation attempts to fill this gap and connect the related econometric and finance literatures which examine the econometric estimation and portfolio performance of multivariate models. We do this by employing a range of economic and Bayesian statistical criteria for performing a comprehensive assessment of the short-horizon predictive ability of different variants of the DCC model for the conditional volatility and correlations of daily FX returns. Our investigation has three distinct objectives. First, we develop a new Bayesian estimation algorithm for the multivariate Asymmetric DCC (ADCC) model of Cappiello, Engle and Sheppard (2006) with either Gaussian or Student-\( t \) innovations. Second, we implement the Bayesian Model Averaging method for forming combined forecasts which exploit information from a large universe of univariate volatility and multivariate correlation models. Finally, we assess the economic value of volatility and corre-
lation timing for the optimal portfolio decision of a risk averse investor implementing a dynamic mean-variance strategy. Our analysis employs daily returns data from five major US dollar nominal spot exchange rates: the UK pound sterling, the Deutsche mark/euro, the Swiss frank, the Japanese yen, and the Canadian dollar, for a sample ranging from January 1976 to December 2006.

An important contribution of our analysis is the use of economic criteria. Statistical evidence of predictability in the dynamic covariance matrix in itself does not guarantee that an investor can earn profits from an asset allocation strategy that exploits this predictability. In practice, ranking models is useful to an investor only if it leads to tangible economic gains. Therefore, we assess the economic value of volatility and correlation timing by evaluating the impact on the performance of dynamic allocation strategies of predictable changes in the conditional covariance matrix of exchange rate returns. We employ mean-variance analysis as a standard measure of portfolio performance and apply quadratic utility, which allows us to quantify how risk aversion affects the economic value of predictability. Our approach builds on empirical studies of volatility timing in stock and exchange rate returns by West, Edison and Cho (1993), Fleming, Kirby, and Ostdiek (2001), Marquering and Verbeek (2004), Han (2006), and Della Corte, Sarno and Tsiakas (2007).

1 Ultimately, we measure how much a risk averse investor is willing to pay for switching from a static portfolio strategy based on the constant covariance model to one that has a dynamic conditional correlation as well as a dynamic conditional volatility specification.

We assess the statistical evidence on covariance predictability in a Bayesian framework. In particular, we design an efficient Bayesian Markov Chain Monte Carlo (MCMC) algorithm for estimating the parameters of the ADCC model with either Gaussian or Student-t innovations. We then rank the competing model specifications by computing the posterior probability of each model. The posterior probability is based on the marginal likelihood and hence it accounts for parameter uncertainty, while imposing a penalty for lack of parsimony (higher dimension). In the context of our Bayesian methodology, an alternative approach to determining the best model available is to form combined forecasts which exploit information from the entire universe of model specifications under consideration. Specifically, we implement the Bayesian Model Averaging (BMA) method, which weighs all conditional volatility and correlation forecasts by the posterior probability of each model. We then compare the BMA results to those obtained from a Deterministic Model Averaging (DMA) strategy, which simply combines all model specifications with equal weights.

The remainder of the paper is organized as follows. In the next section we lay out the multivariate conditional correlation model while Section 3 explains the Bayesian estimation tools and discusses the approach to model selection. Section 4 describes the data, whereas Section 5 discusses the framework for assessing the economic value of volatility and correlation timing for a risk averse investor with a dynamic portfolio allocation strategy. Our empirical results are reported in Section 6. Finally, Section 7 concludes.

2 The Multivariate Framework

2.1 The Dynamic Conditional Correlation (DCC) Model

We adopt the multivariate Dynamic Conditional Correlation (DCC) Model of Engle (2002), which provides a parsimonious and flexible framework for modeling the dynamics of volatilities and correlations, even when the number of assets is very large. Let \( y_t = (y_{1,t}, \ldots, y_{K,t})' \) denote the returns of \( K \) assets at time \( t \):

\[
y_t = \mu_t + \Sigma_t^{1/2} \varepsilon_t,
\]

where \( \mu_t = (\mu_{1,t}, \ldots, \mu_{K,t})' \) is the vector of conditional means, \( \Sigma_t \) is the conditional covariance matrix, \( u_t = y_t - \mu_t \), and \( \varepsilon_t = (\varepsilon_{1,t}, \ldots, \varepsilon_{K,t})' \) are standard normal disturbances.\(^2\) In the DCC model, the conditional covariance matrix is decomposed as follows:

\[
\Sigma_t = D_t R_t D_t
\]

\[
D_t = \text{diag}\{\sigma_{1,t}, \ldots, \sigma_{K,t}\}
\]

\[
R_t = (\text{diag}\{Q_t\})^{-1/2} Q_t (\text{diag}\{Q_t\})^{-1/2}
\]

\[
Q_t = \left( \overline{R} - \Gamma' \overline{R} - \Delta' \overline{R} \Delta \right) + \Gamma' z_{t-1} z_{t-1}' \Gamma + \Delta' Q_{t-1} \Delta,
\]

where \( D_t \) is the \( K \times K \) diagonal matrix of dynamic conditional volatilities, \( R_t \) is the \( K \times K \) dynamic conditional correlation matrix, \( \overline{R} \) is the \( K \times K \) matrix of unconditional correlations, \( Q_t \) is a \( K \times K \) symmetric positive-definite matrix, \( \Gamma \) and \( \Delta \) are \( K \times K \) parameter matrices, and \( z_t = D_t^{-1} u_t \sim N(0, R_t) \). A sufficient condition for \( Q_t \) to be positive definite is that the intercept \( \overline{R} - \Gamma' \overline{R} - \Delta' \overline{R} \Delta \) is positive semi-definite and the initial matrix \( Q_0 \) is positive definite (see Ding and Engle, 2001).

The simple scalar DCC model reduces \( \Gamma = \gamma \) and \( \Delta = \delta \), where \( \{\gamma, \delta\} \) are scalars which are the same for all assets \( i \leq K \). In this case, the typical elements of \( D_t \) and \( R_t \) are \( \sigma_{i,t} \) and \( \rho_{i,j,t} \), respectively, and are defined as follows:

\[
\sigma_{i,t}^2 = \omega_i + \alpha_i u_{it-1}^2 + \beta_i \sigma_{i,t-1}^2
\]

\[
\rho_{i,j,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}}
\]

\[
q_{ij,t} = (1 - \gamma^2 - \delta^2) p_{ij} + \delta^2 \sigma_{i,t-1} \sigma_{j,t-1} + \gamma^2 z_{i,t-1} z_{j,t-1}.
\]

This scalar DCC-GARCH specification assumes: (i) identical correlation parameters \( \{\gamma, \delta\} \) across all assets, (ii) symmetric correlation evolution for matrix \( Q_t \), and (iii) a simple GARCH(1,1) univariate volatility process. In what follows we will generalize the DCC model by relaxing these three assumptions.

\(^2\)For recent studies which generalize the conditional normality assumption see Bauwens and Laurent (2005) and Jondeau and Rockinger (2006).
2.2 The Asymmetric DCC (ADCC) Model

The Asymmetric DCC (ADCC) model of Cappiello, Engle and Sheppard (2006) modifies the dynamic correlation equation as follows:

$$Q_t = (\bar{R} - \Gamma'\bar{R}\Gamma - \Delta'\bar{R}\Delta - \Pi'\bar{P}\Pi) + \Gamma'z_{t-1}z'_{t-1}\Gamma + \Delta'Q_{t-1}\Delta + \Pi'p_{t-1}p'_{t-1}\Pi,$$

(9)

where $\Gamma$, $\Delta$ and $\Pi$ are $K \times K$ parameter matrices, $p_t = I[z_t < 0] \circ z_t$, $I[\bullet]$ is an indicator function taking the value 1 if the argument is true and 0 if otherwise, $\circ$ indicates the Hadamard product, and $\bar{P} = E[p_t p'_t]$. Engle’s (2002) symmetric scalar DCC model is obtained as a special case of the ADCC model when $\Gamma = \delta$, $\Delta = \delta$ and $\Pi = 0$. Another parsimonious variant of the ADCC model results when the matrices $\Gamma$, $\Delta$ and $\Pi$ are assumed to be diagonal. The diagonal ADCC specification will be the most general specification in our analysis.

2.3 The Set of Multivariate Correlation Model Specifications

We isolate the relative importance of dynamic volatility and correlations (symmetric or asymmetric) by estimating the following models:

1. **MLR model**: The static *Multivariate Linear Regression* model with constant variances and covariances. In the MLR model: $\Sigma_t = \Sigma$. This model is equivalent to a multivariate random walk of log-exchange rates with a constant covariance matrix. This has been the benchmark model for forecasting exchange rate returns since the seminal contribution of Meese and Rogoff (1983).

2. **CCC model**: Bollerslev’s (1990) *Constant Conditional Correlation* model with constant correlations and dynamic volatility. In the CCC model: $\Sigma_t = D_t\bar{R}D_t$.

3. **DCC model**: Engle’s (2002) symmetric *Dynamic Conditional Correlation* model with dynamic correlations and dynamic volatility. In the DCC model: $\Sigma_t = D_t R_t D_t$. We estimate both scalar DCC and diagonal $DCC_{diag}$ specifications.

4. **ADCC model**: The Cappiello, Engle and Sheppard (2006) *Asymmetric DCC* model with asymmetric dynamic correlations and dynamic volatility. We estimate both scalar ADCC and diagonal $ADCC_{diag}$ specifications.

2.4 The Set of Univariate GARCH Volatility Specifications

We have so far assumed that a simple GARCH(1,1) univariate volatility process for the DCC model. However, we can specify a number of alternative univariate specifications for the conditional variance, including some of the most popular models in the literature, which we later use in performing Bayesian Model Averaging. The univariate volatility models are the same ones considered by Cappiello, Engle and Sheppard (2006) and are listed below:

1. **GARCH**: Bollerslev (1986):

$$\sigma^2_{t,t} = \omega + \alpha u^2_{t-1} + \beta \sigma^2_{t-1}.$$

\[ \sigma_{i,t} = \omega_i + \alpha_i |u_{i,t-1}| + \beta_i \sigma_{i,t-1}. \]  
(11)


\[ \sigma_t^2_{i,t} = \omega_i + \alpha_i |u_{i,t-1}|^\tau + \beta_i \sigma_t^2_{i,t-1}. \]  
(12)


\[ \ln(\sigma_t^2_{i,t}) = \omega_i + \alpha_i \varepsilon_{i,t-1} + \kappa_i (|\varepsilon_{i,t-1}| - E[|\varepsilon_{i,t}|]) + \beta_i \ln(\sigma_t^2_{i,t-1}); \quad \varepsilon_{i,t-1} = \frac{u_{i,t-1}}{\sigma_{i,t-1}}; \quad E[|\varepsilon_{i,t}|] = \sqrt{\frac{2}{\pi}}. \]  
(13)


\[ \sigma_{i,t} = \omega_i + \alpha_i (|u_{i,t-1}| - \kappa_i u_{i,t-1}) + \beta_i \sigma_{i,t-1}. \]  
(14)

6. GJR-GARCH (Glosten, Jagannathan and Runkle, 1993):

\[ \sigma_t^2_{i,t} = \omega_i + \alpha_i (|u_{i,t-1}| - \kappa_i u_{i,t-1})^2 + \beta_i \sigma_t^2_{i,t-1}. \]  
(15)

7. Asymmetric Power GARCH (APGARCH: Ding, Engle and Granger, 1993):

\[ \sigma_t^2_{i,t} = \omega_i + \alpha_i (|u_{i,t-1}| - \kappa_i u_{i,t-1})^\tau + \beta_i \sigma_t^2_{i,t-1}. \]  
(16)


\[ \sigma_t^2_{i,t} = \omega_i + \alpha_i (u_{i,t-1} + \kappa_i)^2 + \beta_i \sigma_t^2_{i,t-1}. \]  
(17)

9. Nonlinear Asymmetric GARCH (NAGARCH: Engle and Ng, 1993):

\[ \sigma_t^2_{i,t} = \omega_i + \alpha_i (u_{i,t-1} + \kappa_i \sigma_{i,t})^2 + \beta_i \sigma_t^2_{i,t-1}. \]  
(18)

10. tGARCH (Bollerslev, 1987):

\[ \sigma_t^2_{i,t} = \omega_i + \alpha_i u_{i,t-1}^2 + \beta_i \sigma_t^2_{i,t-1}; \quad u_{i,t} \sim t(0,1,\nu_i). \]  
(19)

The news impact curve (Pagan and Schwert, 1990) relates revisions in conditional volatility to the size and sign of shocks. For GARCH, AVGARCH and NARCH, the news impact curve is perfectly symmetric. Whenever \( \kappa_i > 0 \), EGARCH, ZARCH, GJR-GARCH and APGARCH feature a rotated
news impact curve, while AGARCH and NAGARCH employ a shifted news impact curve to achieve asymmetry (see Hentschel, 1995). Although the first nine GARCH specifications are Gaussian, the last one uses Student-\(t\) innovations with independent components \(t(0,1,\nu_i)\), where each asset has a distinct degrees of freedom parameter \(\nu_i\).

In our analysis, we assume a constant conditional mean \(\mu_t = \mu\) for all models. This leads to six multivariate correlation specifications \(\{MLR, CCC, DCC, DCC_{diag}, ADCC\ and\ ADCC_{diag}\}\) and the ten univariate volatility specifications listed above.\(^3\) For example, for the \(ADCC_{diag}\) model with \(tGARCH(1,1)\) volatility we need to estimate three sets of parameters: (i) an \(K \times 3\) vector of GARCH parameters \(\{\omega_i, \alpha_i, \beta_i\}\); (ii) a \(K \times 3\) vector of conditional correlation parameters \(\{\gamma_i, \delta_i, \pi_i\}\), and \(K \times 1\) vector of degrees of freedom of \(\nu_i\). Therefore, \(K = 5\) requires 35 parameter estimates.

3 Estimation and Forecasting

3.1 Bayesian Markov Chain Monte Carlo Estimation

We perform Bayesian estimation of the parameters of all univariate volatility and multivariate correlation model specifications. A distinct advantage of Bayesian inference is that it provides the posterior distribution of the parameters conditional on the data, which holds for finite samples. This posterior distribution can in turn be used as an input in ranking a set of models in a way that accounts for parameter and model uncertainty. We estimate the parameters of (for instance) the \(ADCC_{diag} - tGARCH\) model by designing a new Bayesian MCMC algorithm, which draws some insights from the Bayesian GARCH algorithm of Vrontos, Dellaportas and Politis (2000), and particularly from the Bayesian stochastic volatility (SV) algorithm of Kim, Shephard, and Chib (1998) and Chib, Nardari, and Shephard (2002). The Bayesian MCMC algorithm constructs a Markov chain whose limiting distribution is the target posterior density of the ADCC parameters. The Markov chain is a Gibbs sampler in which all parameters are drawn sequentially from their full conditional posterior distribution. The Gibbs sampler is iterated 5,000 times and the sampled draws, beyond a burn-in period of 1,000 iterations, are treated as variates from the target posterior distribution.

The MCMC algorithm for each of three representative multivariate models \(\{MLR, CCC, ADCC_{diag}\}\) is summarized below. All other specifications are special cases of one of these models. Each algorithm produces estimates of the posterior means \(\theta = \{\mu, \theta_1, \theta_2, \nu\}\), where \(\mu = \{\mu_i\}\) is the parameter vector of the return equation for each series \(i \leq K\), \(\theta_1\) are the volatility parameters of each univariate GARCH process, \(\theta_2\) are the correlation parameters, and \(\nu = \{\nu_i\}\) is a vector with the degrees of freedom parameter of the Student-\(t\) distribution for each return series. For example, for the diagonal \(ADCC_{diag}\) model with \(tGARCH(1,1)\) volatility: \(\theta_1 = \{\omega_i, \alpha_i, \beta_i\}\) and \(\theta_2 = \{\gamma_i, \delta_i, \pi_i\}\). All \(\theta\) parameters are time invariant.

\(^3\)Since our analysis focuses on exchange rates, we have also modeled the dynamic covariance between two dollar exchange rate returns using the dynamic variance of the cross-rate. This is based on triangular no-arbitrage and follows Andersen et al (2003). Despite the simplicity of reducing estimation of multivariate covariances to a set of univariate volatility processes, this model cannot guarantee that (i) that the dynamic covariance matrix is positive definite, and (ii) the correlations are in the \([-1,1]\) range. Hence we have excluded the triangular no-arbitrage model from our analysis.
3.1.1 Prior Specification

For the conditional mean parameters, $\mu = \{\mu_i\}$, we assume a Normal prior $N(\mu, M)$, where $\mu = 0_K$ and $M = I_K$. In the Multivariate Linear Regression model, we define $\Sigma^{-1}$ as the inverse of the covariance matrix (precision matrix) and assume a Wishart prior $W_K(\nu, S)$ with scale matrix $S = I_K$ and degrees of freedom $\nu = K + 2$. These hyperparameter values imply a prior mean $E[\Sigma] = I_K$. Note that the Wishart is an appropriate multivariate distribution for symmetric positive definite matrices.

In general, for the GARCH models we specify $\omega \sim LogN(w, W)$, with $w = -4.8$ and $W = 4$, $\alpha \sim Beta(\alpha, A)$, with $\alpha = 2$ and $A = 40$, $\beta \sim Beta(\beta, P)$, with $\beta = 40$ and $P = 5$, $\kappa \sim U(k, K)$ with $k = -1$ and $K = 1$, and $\tau \sim N(t, T)I_{[0,\infty)}$ with $t = 2$ and $T = 2$. These hyperparameters imply prior means: $E[\omega] = .06$, $E[\alpha] = 0.05$ and $E[\beta] = 0.89$, and hence $E[\sigma^2] = 1$. For the EGARCH, in particular, we use $\omega \sim N(w, W)$ with $w = -0.5$ and $W = 2$, $\alpha \sim N(\alpha, A)$ with $\alpha = 0.1$, $A = 1$, $\kappa \sim N(\kappa, K)$ with $\kappa = 0.2$ and $K = 1$, and $\beta$ is defined in terms of $\beta = 2\beta^* - 1$, where $\beta^* \sim Beta(f, E)$. This implies that the prior on $\beta \in (-1, 1)$ is $p(\beta) = \zeta (0.5(1+\beta))^{p-1}\{0.5(1-\beta)\}^{E-1}$, $f, E > 0.5$, where $\zeta = 0.5\frac{\Gamma(f+E)}{\Gamma(f+1)}$. Specifying $f = 20$ and $E = 1.5$ yields a mean of 0.86 with variance of 0.01.

In the $ADCC_{diag}$ model we need to specify a prior distribution for the diagonal $K \times K$ matrices $\{\Gamma, \Delta, \Pi\}$. Thus we specify multivariate Beta priors $\Gamma \sim Beta(\gamma_0, \Gamma_0)$, $\Delta \sim Beta(\delta_0, \Delta_0)$, and $\Pi \sim Beta(\pi_0, \Pi_0)$. We set the scale matrices $\Gamma_0 = \delta_0 = \Pi_0 = I_K$ and the degrees of freedom parameters $\gamma_0 = \delta_0 = \pi_0 = K + 1$. Finally, for the degrees of freedom parameter vector $\nu = \{\nu_i\}$ we specify a non-informative uniform prior $U[3, 40]$ as in Jacquier, Polson and Rossi (2004). For all models, the hyperparameters are set to reasonable values, but the algorithms described below are robust to the prior specification and initial values.

3.1.2 The Multivariate Linear Regression Algorithm

In the Bayesian Multivariate Linear Regression (MLR) model with Gaussian errors, we need to estimate $\theta = \{\mu, \Sigma^{-1}\}$, where $\mu$ is the vector of conditional means, and $\Sigma^{-1}$ is the constant precision matrix defined as the inverse of the covariance matrix. The simple Gibbs algorithm is summarized below (for more details see Koop, 2003):

1. Initialize $\Sigma^{-1}$.

2. Sample $\mu$ from $y, \Sigma^{-1} \sim N_K(\overline{M}, M)$, where $\overline{M} = [M^{-1} + \Sigma^{-1} \otimes (I'I)]^{-1}$, and $\overline{M} = M^{-1}M + (\Sigma^{-1} \otimes I') vec(y)$.

3. Sample $\Sigma^{-1}$ from $\Sigma^{-1} \sim W_K(\nu, S)$, where $\nu = T + \nu$ and $S = [S^{-1} + (y - \mu)'(y - \mu)]^{-1}$.

4. Go to step 2 and iterate 5,000 times beyond a burn-in of 1,000 iterations.

3.1.3 The CCC Algorithm

In the Constant Conditional Correlation (CCC) model with Gaussian errors (Bollerslev, 1990) we need to estimate $\theta = \{\mu, \theta_1, \theta_2\}$, where $\mu$ is the vector of conditional means, $\theta_1$ are the conditional
variance parameters for each of \( K \) assets, and \( \theta_2 = \{ R^{-1} \} \) are the elements of the unconditional correlation matrix. The algorithm is summarized below:

1. Initialize \( \mu \) and transform the data into \( u_t = (y_t - \mu) \).

2. Sample the variance parameters \( \theta_1 \) from their full conditional posterior density: \( \theta_1 \mid u, \mu, \theta_2 \). This posterior density is not available analytically. We compute the log-likelihood of the transformed data \( u_t \) as function of \( \theta_1 \) (conditional on \( \mu \)) and then we optimize the conditional log-posterior. We generate a proposal from a \( t \) distribution \( t(m, V, \xi) \), where \( m \) is the mode, \( V \) is the scaled inverse of the negative Hessian and \( \xi \) is a tuning parameter. The proposal is then accepted according to the independence chain Metropolis-Hasting algorithm (e.g. Chib and Greenberg, 1995).

3. Sample the elements of \( R^{-1} \) from the Wishart posterior distribution as in Step 3 of the LR algorithm.

4. Sample all the conditional mean coefficients \( \mu \mid y, \theta_1, \theta_2 \) using a precision-weighted average of a set of normal priors and the normal likelihood conditional on \( \theta_1 \) and \( \theta_2 \). Then, update the data \( u_t = (y_t - \mu) \).

5. Go to step 2 and iterate 5,000 times beyond a burn-in of 1,000 iterations.

We design a very similar algorithm for alternative univariate volatility specifications, such as the EGARCH(1,1) model.

### 3.1.4 The ADCC\(_{\text{diag}}\) Algorithm

The ADCC\(_{\text{diag}}\) model with tGARCH(1,1) volatility requires estimation of the parameter matrix \( \theta = \{ \mu, \theta_1, \theta_2, \nu \} \), where \( \mu \) is the vector of the conditional means, \( \theta_1 = \{ \omega_i, \alpha_i, \beta_i \} \) are the GARCH(1,1) parameters for each \( i \leq K \), \( \theta_2 = \{ \Gamma, \Delta, \Pi \} \) are diagonal \( K \times K \) matrices, and \( \nu \) is a vector with the degrees of freedom parameter of the Student-\( t \) distribution for each return series. The algorithm is summarized below:

1. Initialize \( \mu \) and transform the data into \( u_t = (y_t - \mu) \).

2. Sample the conditional variance parameters \( \theta_1 \) from their full conditional posterior density: \( \theta_1 \mid u, \mu, \theta_2 \) following Step 2 of the CCC-GARCH(1,1) algorithm described in section 3.1.3. Use these parameter estimates to form the estimated diagonal volatility matrix \( D_t \) and transform the data \( z_t = D_t^{-1} u_t = R_t^{1/2} \varepsilon_t \).

3. Sample the conditional correlation parameters \( \theta_2 \) from their full conditional posterior density: \( \theta_2 \mid z, \mu, \theta_1 \). This posterior density is not available analytically, and hence we jointly sample from \( p(\Gamma, \Delta, \Pi \mid z) \propto Beta(\gamma_0, \Gamma_0) \times Beta(\delta_0, \Delta_0) \times Beta(\pi_0, \Pi_0) \prod_{t=1}^T N(z_t \mid \Gamma, \Delta, \Pi) \) by implementing the independence-chain Metropolis-Hasting algorithm (e.g. Chib and Greenberg, 1995).
4. Sample all the conditional mean coefficients \( \mu \mid y, \theta_1, \theta_2 \) using a precision-weighted average of a set of normal priors and the normal likelihood conditional on \( \theta_1 \).

5. Update the data \( u_t = (y_t - \mu) \) and use the transformation \( u_t^* = (y_t - \mu) \lambda_t \), where \( \lambda_t \) is a Gamma variable, in order to make \( u_t^* \) conditionally Gaussian. We sample \( \lambda \mid y, \theta \) directly from its posterior:

\[
\lambda_t \mid y_t, \theta \sim \text{Gamma} \left( \frac{\nu + 1}{2}, \frac{2}{(\nu - 2) + (y_t - \mu)^2 / \nu^2} \right).
\]

6. Sample the Student-\( t \) degrees of freedom parameter from the full conditional density: \( \nu \mid u, \theta \).

We optimize the conditional log-posterior with respect to \( \nu \) and then use the mode and a scaled inverse of the negative Hessian to generate a proposal that is accepted according to the independence-chain Metropolis-Hastings algorithm.

7. Go to step 2 and iterate 5,000 times beyond a burn-in of 1,000 iterations.

We design a very similar algorithm for alternative univariate volatility specifications, such as the EGARCH(1,1) model.

### 3.1.5 Numerical Standard Error (NSE)

The mean of the MCMC parameter draws is an asymptotically efficient estimator of the posterior mean of \( \theta \) (see Geweke, 1989). The Numerical Standard Error (NSE) is the square root of the asymptotic variance of the MCMC estimator:

\[
\text{NSE} = \sqrt{\frac{1}{I} \left\{ \hat{\psi}_0 + 2 \sum_{j=1}^{B_I} \text{Ker}(z) \hat{\psi}_j \right\}} \quad (20)
\]

where \( I = 5,000 \) is the number of iterations (beyond the initial burn-in of 1,000 iterations), \( j = 1, \ldots, B_I = 500 \) lags is the set bandwidth, \( z = \frac{j}{B_I} \), and \( \hat{\psi}_j \) is the sample autocovariance of the MCMC draws for each estimated parameter cut according to the Parzen kernel \( \text{Ker}(z) \). The NSE diagnostic is distinct from the MCMC standard deviation. The latter is simply a measure of the variation in the MCMC parameter draws. In contrast, the NSE is a measure of the variation in the posterior mean estimate across many MCMCs we can potentially run. In other words, the NSE measures how much difference we should expect in the estimate of the posterior mean if estimation were to be repeated, and therefore provides a measure of convergence in the Markov chain.

### 3.2 Model Risk and Posterior Probability

Model risk arises from the uncertainty over selecting a model specification. Consistent with our Bayesian approach, a natural statistical criterion for resolving this uncertainty is the posterior probability of each model. Hence, we rank the competing volatility models using the posterior probability, which has three important advantages relative to the log-likelihood: (i) it is based on the marginal likelihood and therefore accounts for parameter uncertainty, (ii) it imposes a penalty for lack of parsimony (higher dimension), and (iii) it forms the basis of the Bayesian Model Averaging strategy discussed below. Ranking the models using the highest posterior probability is equivalent to choosing
the best model in terms of density forecasts and is a robust model selection criterion in the presence of misspecification and non-nested models (e.g. Fernandez-Villaverde and Rubio-Ramirez, 2004).

Consider a set of $N$ models $M_1, ..., M_N$. We form a prior belief $\pi(M_i)$ on the probability that the $i$th model is the true model, observe the returns data $\{y_t\}$, and then update our belief that the $i$th model is true by computing the posterior probability of each model as follows:

$$p(M_i | y) = \frac{p(y | M_i) \pi(M_i)}{\sum_{j=1}^{N} p(y | M_j) \pi(M_j)} \quad (21)$$

where $p(y | M_i)$ is the marginal likelihood of the $i$th model defined as:

$$p(y | M_i) = \int_{\theta} p(y, \theta | M_i) d\theta = \int_{\theta} p(y | \theta, M_i) \pi(\theta | M_i) d\theta \quad (22)$$

In Equation 21 above we set our prior belief to be that all models are equally likely, i.e. $\pi(M_i) = \frac{1}{N}$.

Note that the marginal likelihood is an averaged (not a maximized) likelihood. This implies that the posterior probability is an automatic “Occam’s Razor” in that it integrates out parameter uncertainty. Furthermore, the marginal likelihood is simply the normalizing constant of the posterior density and (suppressing the model index for simplicity) it can be written as:

$$p(y) = \frac{f(y | \theta) \pi(\theta)}{\pi(\theta | y)} \quad (23)$$

where $f(y | \theta)$ is the likelihood, $\pi(\theta)$ the prior density of the parameter vector $\theta$, $\pi(\theta | y)$ the posterior density, and $\theta$ is evaluated at the posterior mean. Since $\theta$ is drawn in the context of MCMC sampling, the posterior density $\pi(\theta | y)$ is computed using the technique of reduced conditional MCMC runs of Chib (1995). For the $\theta_2$ parameters of each univariate GARCH specification sampled in the MCMC chain by implementing a Metropolis-Hastings algorithm, the posterior density is computed as in Chib and Jeliazkov (2001).

### 3.3 Combined Forecasts

Assessing the predictive ability of multivariate correlation and univariate volatility models primarily involves a pairwise comparison of the competing models. However, given that we do not know which one of the models is true, it is important that we assess the performance of combined forecasts proposed by the seminal work of Bates and Granger (1969). Specifically, we design three strategies based on a combination of forecasts for both the conditional volatility and correlation of exchange rate returns: the Deterministic Model Average (DMA) strategy, the Bayesian Model Average (BMA) strategy, and the Bayesian Winner (BW) strategy.

---

4 The information one can extract from the posterior probability of a model is similar to using the Kullback-Leibler Information Criterion (KLIC). Specifically, Fernandez-Villaverde and Rubio-Ramirez (2004) show that choosing the model with the highest posterior probability is equivalent to selecting the best model under the KLIC. This is an attractive feature of our Bayesian approach because there is a complete axiomatic foundation that justifies why KLIC is the best criterion a rational agent should use in choosing between models (e.g. Csiszar, 1991).

5 Occam’s Razor is the principle of parsimony, which states that among two competing theories that make exactly the same prediction, the simpler one is best.

We assess the economic value of combined forecasts by treating the $\text{DMA}$, $\text{BMA}$ and $\text{BW}$ strategies the same way as any of the individual correlation or volatility models. For instance, we compute the performance fee, $\Phi$, for the $\text{BMA}$ one-step ahead forecasts of the conditional mean, volatility and correlation and then compare them to the Multivariate Linear Regression benchmark. We perform this exercise for the universe of all models under consideration.

Consequently, our empirical analysis of correlation and volatility timing further contributes to the literature by incorporating both a statistical view of Bayesian parameter uncertainty and an economic view of the effect of model uncertainty on asset allocation decisions and performance. In contrast to Avramov (2002), however, our approach does not attempt to separate the effects of parameter and model uncertainty. Finally, we only consider model uncertainty within the large (but finite) universe of the models discussed in Section 2.

### 3.3.1 The $\text{DMA}$ Strategy

Quite simply, the $\text{DMA}$ strategy involves taking an equally weighted average of the conditional volatility and correlation forecasts from a given universe of available models. Hence, for a set of $N$ models the $\text{DMA}$ strategy is referred to as the $1/N$ strategy. Since this is a strategy that does not require period-by-period updating of the weights in the forecast combination, it can be readily evaluated in-sample and out-of-sample on the basis of conditioning information available at the time of the forecast.

### 3.3.2 The $\text{BMA}$ Strategy

In the context of our Bayesian approach, it is natural to implement the $\text{BMA}$ method originally discussed in Leamer (1978) and surveyed in Hoeting, Madigan, Raftery and Volinsky (1999). The $\text{BMA}$ strategy accounts directly for uncertainty in model selection, and is in fact easy to implement once we have the output from the MCMC simulations. Define $f_{i,t}$ as the forecast density of each of the $N$ competing models at time $t$. Then, the $\text{BMA}$ forecast density is given by:

$$f_{t}^{\text{BMA}} = \sum_{i=1}^{N} p_t(M_i \mid y_t) f_{i,t}$$

(24)

where $p_t(M_i \mid y_t)$ is the posterior probability of model $M_i$ given the data $y_t$.

It is important to note that the $\text{BMA}$ weights vary not only across models but also across time periods as does the marginal predictive density (and hence marginal likelihood) of each model. In particular, at each time period we estimate the one-step ahead predictive density $f_t(y_t \mid \mathcal{F}_{t-1}, \theta)$ and the posterior density $\pi_t(\theta \mid y_t)$. We can then compute the time-varying marginal likelihood using Equation 23, and insert it into Equation 21 to finally calculate the posterior probability of each model at each time period. It is crucial to emphasize that we evaluate the $\text{BMA}$ strategy ex-ante. We do this by lagging the posterior probability of each model for the following reason. Suppose that we need to compute the period-$t$ $\text{BMA}$ forecasts of the conditional volatility and correlation for the five currencies we include in the FX portfolio. Knowing the volatility and correlation forecasts implied by each model for the stock returns is not sufficient. We also need the realized data point $y_t$ in order to evaluate the predictive density $f_t(y_t \mid \mathcal{F}_{t-1}, \theta)$. Since the realized data point $y_t$ is only observed
ex post, the only way to form the BMA weights ex ante is to lag the predictive density and thus use $f_{t-1}(y_{t-1} | \mathcal{F}_{t-2}, \theta)$. The same method is applied both in-sample and out-of-sample.

### 3.3.3 The BW Strategy

Under the BW strategy, in each time period we select the set of one-step ahead conditional volatility and correlation from the empirical model that has the highest marginal predictive density in that period. In other words, the BW strategy only uses the forecasts of the “winner” model in terms of marginal likelihood, and hence discards the forecasts of the rest of the models. Clearly, there is no model averaging in the BW strategy. Similar to the BMA, the BW strategy is evaluated ex ante using the lagged predictive marginal densities.

### 4 Data and Descriptive Statistics

Our analysis employs daily returns data from five major US dollar nominal spot exchange rates for the period of January 1976 to December 2006 corresponding to a total of 8069 observations. The exchange rates are taken from EcoWin and include the UK pound sterling (USD/GBP), the Deutsche mark/euro (USD/EUR), the Swiss frank (USD/CHF), the Japanese yen (USD/JPY), and the Canadian dollar (USD/CAD). After the introduction of the Euro in January 1999, we use the official Euro-Deutsche mark conversion rate to obtain the USD/EUR series.

Table 2 reports the descriptive statistics for the daily percent FX returns. For our sample period, the means are near zero ranging from $-0.0017$ for USD/CAD to 0.0117 for USD/JPY. The daily volatilities revolve between 0.326 for USD/CAD to 0.736 for USD/CHF. Skewness is negative for three of the five currency returns, while kurtosis ranges from 6.02 for USD/EUR to 9.78 for USD/GBP. Finally, Panel B of Table 2 shows the correlation matrix for the five FX returns, which range between 0.120 and 0.819.

### 5 Measuring the Economic Value of Volatility and Correlation Risk

This section discusses the framework we use in order to evaluate the impact of predictable changes in the volatility and correlations of asset returns on the performance of dynamic allocation strategies.

#### 5.1 Quadratic Utility

Mean-variance analysis is a natural framework for assessing the economic value of strategies which exploit predictability in the mean and the covariance matrix. In particular, we rank the performance of competing multivariate models using the West, Edison and Cho (1993) methodology, which is based on mean-variance analysis with quadratic utility. The investor’s realized utility in period $t+1$ can be written as:

$$U(W_{t+1}) = W_{t+1} - \frac{\lambda}{2} W_{t+1}^2 = W_t R_{p,t+1} + \frac{\lambda}{2} W_{t+1}^2 R_{p,t+1}^2,$$  

(25)

where $W_{t+1}$ is the investor’s wealth at $t + 1$, $\lambda$ determines her risk preference, and

$$R_{p,t+1} = 1 + r_{p,t+1} = 1 + (1 - w^f_t) r_f + w^f_t y_{t+1}$$  

(26)
is the period \( t + 1 \) gross return on her portfolio.

We quantify the economic value of volatility and correlation timing by setting the investor’s degree of relative risk aversion (RRA) \( \delta_t = \lambda W_t / (1 - \lambda W_t) \) equal to a constant value \( \delta \). In this case, West, Edison and Cho (1993) demonstrate that one can use the average realized utility, \( \bar{U}(\cdot) \), to consistently estimate the expected utility generated by a given level of initial wealth. Specifically, the average utility for an investor with initial wealth \( W_0 \) is equal to:

\[
\bar{U}(\cdot) = W_0 \sum_{t=0}^{T-1} \left\{ R_{p,t+1} - \frac{\delta}{2(1 + \delta)} R^2_{p,t+1} \right\}. \tag{27}
\]

We standardize the investor problem by assuming she allocates $1 in every time period.

Average utility depends on taste for risk. As in West, Edison and Cho (1993) and Fleming, Kirby and Ostdiek (2001), fixing \( \delta \) implies that expected utility is linearly homogeneous in wealth: double wealth and expected utility doubles. Furthermore, by fixing \( \delta \) rather than \( \lambda \), we are implicitly interpreting quadratic utility as an approximation to a non-quadratic utility function with the approximating choice of \( \lambda \) dependent on wealth. The estimate of expected quadratic utility given in Equation 27 is used to implement the Fleming, Kirby and Ostdiek (2001) framework for assessing the economic value of our strategies in the context of dynamic asset allocation.

A critical aspect of mean-variance analysis is that it applies exactly only when the return distribution is normal or the utility function is quadratic. Hence, the use of quadratic utility is not necessary to justify mean-variance optimization. For instance, one could instead consider using utility functions belonging to the constant relative risk aversion (CRRA) class, such as power or log utility. However, quadratic utility is an attractive assumption because it allows us to consider non-normal distributions of returns, while remaining within the mean-variance framework as well as providing a high degree of analytical tractability.\(^7\) Additionally, quadratic utility may be viewed as a second order Taylor series approximation to expected utility. In an investigation of the empirical robustness of the quadratic approximation, Hlawitschka (1994) finds that a two-moment Taylor series expansion “may provide an excellent approximation” (p. 713) to expected utility and concludes that the ranking of common stock portfolios based on two-moment Taylor series is “almost exactly the same” (p. 714) as the ranking based on a wide range of utility functions.

### 5.2 Optimal Portfolio Choice in Dynamic Mean-Variance

Constructing the optimal portfolio weights requires estimates of the conditional expected returns, variances and covariances. We consider the four classes of multivariate models for conditional correlations discussed in Section 2.3 and the ten conditional volatility models listed in Section 2.4. Since we are primarily interested in the relative impact of volatility and correlation timing, all models assume a constant mean for exchange rate returns. By design, in this setting the optimal weights will vary across models only to the extent that forecasts of the conditional volatility and correlations will vary, which is precisely what the empirical models provide. The benchmark against which we compare the model specifications is the static-covariance Multivariate Linear Regression model

\(^7\)In fact, assuming quadratic utility allows us to use the Fleming, Kirby and Ostdiek (2001) framework (also based on quadratic utility) for evaluating the performance of fat-tailed volatility specifications, such as the \( tGARCH \) model of Bollerslev (1987).
In short, our objective is to determine whether in FX markets there is economic value in (i) conditioning on dynamic volatility, (ii) conditioning on dynamic correlations, and (iii) if so, which volatility and correlation specification works best.

Consider an investor who has a one-day horizon and constructs a dynamically rebalanced portfolio. Computing the time-varying weights of this portfolio requires one-step ahead forecasts of the conditional mean and the conditional variance-covariance matrix. Let \( y_{t+1} \) denote the vector of risky asset returns; \( \mu_{t+1} \) is the conditional expectation of \( y_{t+1} \); and \( \Sigma_{t+1|t} \) is the conditional variance-covariance matrix of \( y_{t+1} \). In what follows, we will consider three rules for optimal dynamic asset allocation: maximum expected utility, maximum expected return and minimum volatility.

### 5.2.1 Maximum Expected Utility

The maximum expected utility rule leads to the optimal portfolio on the efficient frontier. At each period \( t \), the investor solves the following problem:

\[
\max_{w_t} \left\{ E[ U(W_{t+1})] = \mu_{p,t+1} - \frac{\lambda}{2} \sigma_{p,t+1}^2 \right\} \\
\text{s.t.} \quad \mu_{p,t+1} = w_t' \mu_{t+1|t} + (1 - w_t' \mathbf{1}) r_f \\
\quad \sigma_{p,t+1}^2 = w_t' \Sigma_{t+1|t} w_t,
\]

where \( w_t \) is the \( K \times 1 \) vector of portfolio weights on the risky assets; \( \mu_{p,t+1} \) is the conditional expected return of the portfolio; \( \sigma_{p,t+1}^2 \) is the conditional volatility of the portfolio returns; and \( r_f \) is the return on the riskless asset. The solution to this optimization problem delivers the risky asset weights:

\[
w_t = \frac{1}{\lambda} \Sigma_{t+1|t}^{-1} \left( \mu_{t+1|t} - \lambda r_f \right).
\]

The weight on the riskless asset is \( 1 - w_t' \mathbf{1} \).

### 5.2.2 Maximum Expected Return

The maximum expected return rule leads to a portfolio allocation on the efficient frontier for a given target conditional volatility. At each period \( t \), the investor solves the following problem:

\[
\max_{w_t} \left\{ \mu_{p,t+1} = w_t' \mu_{t+1|t} + (1 - w_t' \mathbf{1}) r_f \right\} \\
\text{s.t.} \quad \left( \sigma_p^2 \right)^{\frac{1}{2}} = w_t' \Sigma_{t+1|t}^{\frac{1}{2}} w_t,
\]

where \( \sigma_p^2 \) is the target conditional volatility of the portfolio returns. The solution to the maximum expected return rule gives the following risky asset weights:

\[
w_t = \frac{\sigma_p^2}{\sqrt{C_t}} \Sigma_{t+1|t}^{-1} \left( \mu_{t+1|t} - \lambda r_f \right),
\]

where \( C_t = \left( \mu_{t+1|t} - \lambda r_f \right)' \Sigma_{t+1|t}^{-1} \left( \mu_{t+1|t} - \lambda r_f \right) \).
5.2.3 Minimum Volatility

The minimum volatility rule leads to the best mean-variance allocation for a given target expected return. At each period $t$, the investor solves the following problem:

$$\min_{w_t} \sigma_{p,t+1}^2 = w_t' \Sigma_{t+1|t} w_t$$

s.t. $\mu_p^* = w_t' \mu_{t+1|t} + (1 - w_t') r_f^2$, \quad (35)

where $\mu_p^*$ is the target expected portfolio return of the portfolio returns. The solution to the minimum volatility rule is as follows:

$$w_t = \frac{(\mu_p^* - r_f)}{(\mu_t - r_f)} \Sigma_t^{-1} \frac{(\mu_t - r_f)}{(\mu_t - r_f)}$$

s.t. $\mu_p^* = w_t' \mu_{t+1|t} + (1 - w_t') r_f^2$, \quad (36)

5.3 Performance Measures

At any point in time, one set of estimates of the conditional mean and variance is better than a second set if investment decisions based on the first set lead to higher average realized utility, $U$. Alternatively, the optimal model requires less wealth to yield a given level of $U$ than a suboptimal model. Following Fleming, Kirby and Ostdiek (2001) we measure the economic value of our multivariate strategies by equating the average utilities for selected pairs of portfolios. Suppose, for example, that holding a portfolio constructed using the optimal weights based on the MLR model yields the same average utility as holding the ADCC$_{diag}$ optimal portfolio that is subject to daily expenses $\Phi$, expressed as a fraction of wealth invested in the portfolio. Since the investor would be indifferent between these two strategies, we interpret $\Phi$ as the maximum performance fee she will pay to switch from the MLR to the ADCC$_{diag}$ strategy. In other words, this utility-based criterion measures how much a mean-variance investor is willing to pay for conditioning on dynamic volatility and correlation forecasts. The performance fee will depend on the investor’s degree of relative risk aversion. To estimate the fee, we find the value of $\Phi$ that satisfies:

$$\sum_{t=0}^{T-1} \left\{ (R_{p,t+1}^* - \Phi) - \frac{\delta}{2(1 + \delta)} (R_{p,t+1}^* - \Phi)^2 \right\} = \sum_{t=0}^{T-1} \left\{ R_{p,t+1} - \frac{\delta}{2(1 + \delta)} R_{p,t+1}^2 \right\},$$

where $R_{p,t+1}^*$ is the gross portfolio return constructed using the expected return, volatility and correlation forecasts from the ADCC$_{diag}$ model, and $R_{p,t+1}$ is the gross portfolio return implied by the benchmark MLR model.

In the context of mean-variance analysis, a commonly used measure of economic value is the Sharpe ratio. However, as suggested by Marquering and Verbeek (2004) and Han (2006), the Sharpe ratio can be misleading because it severely underestimates the performance of dynamic strategies. Specifically, the realized Sharpe ratio is computed using the sample standard deviation of the realized portfolio returns and hence it overestimates the conditional risk an investor faces at each point in time. Furthermore, the Sharpe ratio cannot quantify the exact economic gains of the dynamic strategies over the static covariance strategy in the direct way of the performance fees. Therefore, our economic analysis of short-horizon exchange rate predictability focuses primarily on performance fees.
Finally, we also compute the $M^2$ measure of Modigliani and Modigliani (1997) which evaluates the abnormal return a dynamic strategy would have earned if it had the same risk as the static benchmark. The $M^2$ measure is defined as follows and is directly related to the Sharpe ratio:

$$M^2 = \frac{\sigma_b}{\sigma_p} (\mu_p - r_f) - (\mu_b - r_f) = \sigma_b (SR_p - SR_b),$$

where $\sigma_b$ and $SR_b$ are the volatility and Sharpe ratio of the benchmark static strategy.

### 5.4 The Dynamic Strategies

In this mean-variance quadratic-utility framework, we design an FX strategy which involves trading the five major currencies under investigation. Consider a US investor who builds a portfolio by allocating his wealth between six bonds: one domestic (US), and five foreign bonds (Canada, UK, Germany, Switzerland and Japan). At the beginning of each day, the six bonds yield a riskless return in local currency. Hence the only risk the US investor is exposed to is FX risk. Each day the investor takes two steps. First, she uses each of the models to forecast the one-day ahead conditional mean, volatility and correlations of the exchange rate returns. Second, conditional on the forecasts of each model, she dynamically rebalances her portfolio by computing the new optimal weights for each of the maximum utility, maximum return and minimum volatility strategies. This setup is designed to inform us whether using one particular conditional volatility and correlation specification affects the performance of a short-horizon allocation strategy in an economically meaningful way. The yields of the riskless bonds are proxied by daily Eurodeposit rates. Following Marquering and Verbeek (2004) and Han (2006) we set $\delta = 6$. We report the estimates of $\Phi$ and $M^2$ as annualized fees in basis points.

### 5.5 Transaction Costs

The impact of transaction costs is an essential consideration in assessing the profitability of trading strategies. This is especially true in our case because the trading strategy based on the $MLR$ benchmark is static, whereas the $CCC$, $DCC$ and $ADCC$ models generate dynamic strategies. Therefore, we calculate the performance fees of the dynamic strategies relative to the static benchmark for the case when the proportional transaction cost to the portfolio return is equal to 0, 1 and 2 basis points per day. In foreign exchange trading this is a realistic range for the level of transaction costs as it roughly corresponds to 1 or 2 pips per currency unit. Following Marquering and Verbeek (2004) by deducting the proportional transaction cost from the net portfolio return ex post. This ignores the fact that mean-variance portfolios are no longer optimal in the presence of transaction costs but maintains the simplicity and tractability of mean-variance analysis.

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8 Due to lack of data availability, we derive the daily yield of each bond using daily data from the one-month ahead Eurocurrency deposits.

9 The multivariate linear regression model ($MLR$) is the only empirical model that assumes constant mean and covariance matrix. Therefore, the in-sample optimal weights for the $MLR$ trading strategy remain constant over time. However, the out-of-sample optimal weights will vary because every month we re-estimate the drift and covariance matrix of the $MLR$ model.

10 The typical transaction cost a large investor pays in the FX market is 1 pip, which is equal to 0.01 cent. For example if the GBP/USD exchange rate is equal to 2.0000, 1 pip would raise it to 2.0001 and this would correspond to 1/2 basis point proportional cost.
We can avoid these concerns by calculating the break-even transaction cost, $\tau^{BE}$, that renders investors indifferent between two strategies (e.g. Han, 2006). In particular, we assume that transaction costs equal a fixed proportion ($\tau$) of the value traded in each asset: $\tau |w_t - w_{t-1} \frac{1+r_t}{1+r_{p,t}}|$. In comparing a dynamic strategy with the static strategy, an investor who pays transaction costs lower than $\tau^{BE}$ will prefer the dynamic strategy. We report $\tau^{BE}$ in daily basis points.\footnote{In contrast to $\Phi$ and $M2$, which are reported in annual basis points, $\tau^{BE}$ is reported in daily basis points because $\tau^{BE}$ is a proportional cost paid every day when the portfolio is rebalanced.}

6 Empirical Results

6.1 Estimation of Multivariate Models

We begin our statistical evaluation of short-horizon volatility and correlation timing by performing Bayesian estimation of the parameters for the set of candidate multivariate models. Table 2 presents the posterior mean estimates for the parameters of the $ADCC_{diag}$ model with GARCH(1,1) innovations. The table demonstrates that volatilities and correlations are highly persistent for all exchange rate returns. The persistence parameters are similar across assets and are highly statistically significant. Furthermore, the parameter $\pi$ indicating asymmetry in dynamic correlations is close to zero for all exchange rates with the exception of the UK pound. Note that in our Bayesian MCMC framework we assess statistical significance by computing the Numerical Standard Error ($NSE$) as defined in Section 3.1.5.

6.2 Evaluating Forecasts Using Statistical Criteria

We assess the statistical evidence on short-horizon exchange rate volatility and correlation timing by ranking the large set of competing models according to their log-likelihood. The log-likelihood values shown in Table 3 lead to the following conclusions: (i) all constant correlation models with dynamic volatility are better than the static benchmark; (ii) all dynamic correlation models are better than constant correlation models; (iii) among CCC models, the best performing volatility specifications are EGARCH and APGARCH. Even though the differences across volatility specifications are small, the standard GARCH specification is the worst performer; (iv) the diagonal DCC is always better than the scalar DCC; and finally (v) the symmetric DCC is always better than the asymmetric DCC. In short, the best model overall is the diagonal symmetric DCC with APGARCH volatility.

6.3 Evaluating Forecasts Using Economic Criteria

We assess the economic value of short-horizon exchange rate volatility and correlation timing by analyzing the performance of the dynamically rebalanced portfolios constructed using the set of candidate multivariate models. Our analysis focuses on the performance fee, $\Phi$, a US investor is willing to pay for switching from one FX strategy to another. The fees are reported in Table 4, which displays the economic value of each volatility and correlation specification relative to the benchmark multivariate linear regression model ($MLR$).

Panel A of Table 7 shows that the maximum utility performance fee for switching from $MLR$ to the constant conditional correlation with dynamic $GARCH$ volatility ($CCC - GARCH$) is a
staggering 494 annual basis points (bps). The fee for switching from MLR to the dynamic conditional correlation with dynamic GARCH volatility (DCC – GARCH) rises to 865 bps. This is clear evidence that in addition to the well-known result that there is substantial economic value associated with volatility timing, there is also high economic value specifically due to timing dynamic correlations. This is a novel finding in this literature.

The performance fees are fairly robust to transaction costs; for a proportional daily transaction cost of 1 bps (2 bps) they slightly decrease to 429 (363) bps for CCC – GARCH and 766 (667) bps for DCC – GARCH. These results are also reflected by the rise in the Sharpe ratio from 1.33 for MLR to 1.64 for CCC – GARCH and to 1.86 for DCC – GARCH. We can conclude, therefore, that there is substantial and distinct economic value not only in volatility but also in correlation timing for daily FX strategies. It is equally important that the economic value results remain unaffected by different volatility specifications, diagonal correlation structure or asymmetric correlations. Therefore, a simple scalar DCC model with GARCH innovations can capture all the gains from dynamic volatility and correlations for the purpose of dynamic asset allocation across currencies.

The maximum return and minimum volatility strategies display similar results. The fees are a bit less pronounced than for maximum utility but this depends crucially on the target volatility or target expected return that we set. In our calculations we follow Fleming, Kirby and Ostdiek (2001), Marquering and Verbeek (2004) and Han (2006) in setting $\sigma_p = 12\%$ for the expected return strategy, $\mu_p = 10\%$ for the minimum volatility strategy, and a degree of relative risk aversion $\delta = 6$.

If transaction costs are sufficiently high, the period-by-period fluctuations in the dynamic weights of an optimal strategy will render the strategy too costly to implement relative to the static covariance model. In addition to computing performance fees net of transaction costs, we address this concern by computing the break-even transaction cost, $\tau^{BE}$, as the minimum daily proportional cost which cancels out the utility advantage (and hence positive performance fee) of a given strategy. In comparing a dynamic strategy with the static MLR strategy, an investor who pays a transaction cost lower than $\tau^{BE}$ will prefer the dynamic strategy. The $\tau^{BE}$ values are expressed in daily basis points. In general, Table 4 shows that the $\tau^{BE}$ values revolve around 7 bps for constant conditional correlation models and between 8 – 10 bps for dynamic correlation models. Given that the proportional cost of portfolio rebalancing in FX markets is around 1 or 2 pips, we can conclude that the economic value of dynamic volatility and correlation is robust to reasonable transaction costs.

7 Conclusion

The development of tractable multivariate volatility models and the economic evaluation of covariance forecasts in a dynamic allocation framework are two related lines of research in econometrics and finance. The main objective of this paper is to bridge these two strands of financial econometrics literature and assess the relative economic value of volatility and correlation timing. We do so by focusing on the foreign exchange market and estimating a large universe of multivariate models based on the DCC model of Engle (2002). In the process, we contribute to the econometric literature by designing a new Bayesian MCMC algorithm for efficient estimation of the DCC model. The Bayesian approach allows us to explicitly account for parameter and model uncertainty in our statistical and economic evaluation. More importantly, we contribute to the growing literature on
exchange rate predictability and volatility timing by demonstrating that there is high economic value in timing correlations, in addition to the substantial economic value in volatility timing. This result is robust to reasonable transaction costs, which in FX trading are generally low. Finally, we find that the simple DCC model with GARCH volatility performs equally well as models with alternative volatility specifications, diagonal correlation structure and asymmetric correlations. Despite its simplicity, therefore, the multivariate DCC model is a powerful instrument in the practice of currency management.
Table 1
Descriptive Statistics for Daily Exchange Rate Returns

Panel A: Percent Returns

<table>
<thead>
<tr>
<th></th>
<th>USD/GBP</th>
<th>USD/EUR</th>
<th>USD/CHF</th>
<th>USD/JPY</th>
<th>USD/CAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>−0.0004</td>
<td>0.0070</td>
<td>0.0095</td>
<td>0.0117</td>
<td>−0.0017</td>
</tr>
<tr>
<td>STD</td>
<td>0.620</td>
<td>0.653</td>
<td>0.736</td>
<td>0.664</td>
<td>0.326</td>
</tr>
<tr>
<td>Min</td>
<td>−7.59</td>
<td>−5.87</td>
<td>−5.83</td>
<td>−4.15</td>
<td>−2.22</td>
</tr>
<tr>
<td>Max</td>
<td>4.64</td>
<td>4.14</td>
<td>5.17</td>
<td>6.40</td>
<td>2.00</td>
</tr>
<tr>
<td>Skewness</td>
<td>−0.356</td>
<td>−0.005</td>
<td>0.083</td>
<td>0.609</td>
<td>−0.087</td>
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<td>1964</td>
<td>1936</td>
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Panel B: Correlation Matrix

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<th>USD/CHF</th>
<th>USD/JPY</th>
<th>USD/CAD</th>
</tr>
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<td>0.336</td>
<td>0.162</td>
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<td>0.819</td>
<td>0.491</td>
<td>0.165</td>
</tr>
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<td>USD/CHF</td>
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<td>1.00</td>
<td>0.543</td>
<td>0.169</td>
</tr>
<tr>
<td>USD/JPY</td>
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<td>0.491</td>
<td>0.543</td>
<td>1.00</td>
<td>0.120</td>
</tr>
<tr>
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<td>0.165</td>
<td>0.169</td>
<td>0.120</td>
<td>1.00</td>
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The table summarizes the descriptive statistics for the daily percent exchange rate returns. The data sample range from January 2, 1976 through December 29, 2006 for a sample size of 8,069 daily observations. BL(10) denotes the Box-Ljung Q-statistic at 10 lags.
Table 2
Posterior Means for the ADCC$_{diag}$-GARCH(1,1) Model

<table>
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<tr>
<th>FX Rates</th>
<th>$\mu$</th>
<th>$\omega$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
<th>$\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD/GBP</td>
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<td>0.0064</td>
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<td>0.915</td>
<td>0.145</td>
<td>0.983</td>
<td>0.123</td>
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<tr>
<td></td>
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<td>(9×10^{-6})</td>
<td>(5×10^{-5})</td>
<td>(6×10^{-5})</td>
<td>(4×10^{-5})</td>
<td>(8×10^{-6})</td>
<td>(7×10^{-5})</td>
</tr>
<tr>
<td>USD/EUR</td>
<td>0.0071</td>
<td>0.0042</td>
<td>0.065</td>
<td>0.928</td>
<td>0.192</td>
<td>0.979</td>
<td>0.047</td>
</tr>
<tr>
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<td>(3×10^{-5})</td>
<td>(6×10^{-6})</td>
<td>(5×10^{-5})</td>
<td>(5×10^{-5})</td>
<td>(3×10^{-5})</td>
<td>(7×10^{-6})</td>
<td>(6×10^{-5})</td>
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<td>USD/CHF</td>
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<td>0.0053</td>
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<td>(9×10^{-6})</td>
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<tr>
<td>USD/JPY</td>
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<td>0.926</td>
<td>0.145</td>
<td>0.986</td>
<td>0.039</td>
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<td>(9×10^{-6})</td>
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<td>(6×10^{-5})</td>
<td>(4×10^{-5})</td>
<td>(8×10^{-6})</td>
<td>(7×10^{-5})</td>
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<td>USD/CAD</td>
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The table presents the Bayesian MCMC estimates for the posterior means of the diagonal asymmetric $ADCC_{diag}$-GARCH(1,1) model parameters applied on daily percent exchange rate returns. The numbers in parenthesis indicate the Numerical Standard Error (NSE).
<table>
<thead>
<tr>
<th>Constant Variance</th>
<th>Static Benchmark</th>
<th>Volatility Timing</th>
<th>Volatility and Correlation Timing</th>
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<td>CCC</td>
<td>DCC</td>
</tr>
<tr>
<td>\textit{GARCH}</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<td>−21993</td>
<td>−21877</td>
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<tr>
<td>\textit{NARCH}</td>
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<td>−22053</td>
<td>−21954</td>
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<tr>
<td>\textit{EGARCH}</td>
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<td>−21959</td>
<td>−21853</td>
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<tr>
<td>\textit{ZARCH}</td>
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<td>−21953</td>
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<td>\textit{GJR – GARCH}</td>
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<td>\textit{APGARCH}</td>
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<td>\textit{AGARCH}</td>
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<td>\textit{NAGARCH}</td>
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<td>−21959</td>
<td>−21844</td>
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<td></td>
<td>−24535</td>
<td>−21955</td>
<td>−21839</td>
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</table>

The table presents the log-likelihood values for the multivariate linear regression (MLR), constant conditional correlation (CCC), scalar and diagonal dynamic conditional correlation (DCC), and scalar and diagonal asymmetric DCC (ADCC) models for each of ten univariate GARCH specifications.
Table 4
Portfolio Performance with Daily Rebalance

Panel A: Maximum Expected Utility Strategy ($\delta = 6$)

<table>
<thead>
<tr>
<th>Model</th>
<th>$\mu_p$</th>
<th>$\sigma_p$</th>
<th>$SR$</th>
<th>$\Phi_{(TC=0)}$</th>
<th>$\Phi_{(TC=1)}$</th>
<th>$\Phi_{(TC=2)}$</th>
<th>$M2$</th>
<th>$\tau^{BE}$</th>
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</tr>
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<td>429</td>
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<td>375</td>
<td>7.55</td>
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<tr>
<td><strong>Volatility Timing</strong></td>
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<tr>
<td>CCC – GARCH</td>
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<td>13.5</td>
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<td>375</td>
<td>7.55</td>
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<td>345</td>
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<td>459</td>
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(continued)
Table 4 (continued)

Portfolio Performance with Daily Rebalance

Panel B: Maximum Expected Return Strategy ($\sigma^*_p = 12\%$, $\delta = 6$)

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<th>Model</th>
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<th>$\Phi^{(TC=1)}$</th>
<th>$\Phi^{(TC=2)}$</th>
<th>$M_2$</th>
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<td>684</td>
<td>9.65</td>
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</table>

(continued)
Table 4 (continued)
Portfolio Performance with Daily Rebalance

<table>
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<th>Panel C: Minimum Volatility Strategy ($\mu^*_p = 10%, \delta = 6$)</th>
<th>$\mu_p$</th>
<th>$\sigma_p$</th>
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<th>$\Phi_{(TC=1)}$</th>
<th>$\Phi_{(TC=2)}$</th>
<th>$M2$</th>
<th>$\tau^{BE}$</th>
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<td>101</td>
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<td>5.84</td>
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<td>69</td>
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<td>12.5</td>
<td>0.59</td>
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</tr>
<tr>
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<td>15.4</td>
<td>11.6</td>
<td>0.75</td>
<td>345</td>
<td>314</td>
<td>284</td>
<td>345</td>
<td>11.4</td>
</tr>
<tr>
<td>ADCC$_\text{diag}$ – GARCH</td>
<td>15.3</td>
<td>11.5</td>
<td>0.74</td>
<td>334</td>
<td>302</td>
<td>271</td>
<td>333</td>
<td>10.6</td>
</tr>
</tbody>
</table>

The table reports on the performance of multivariate models with dynamic volatility and correlation against the benchmark model with static covariance matrix. MLR is the multivariate linear regression model, CCC stands for constant conditional correlation and DCC is dynamic conditional correlation. The annualized percent mean, volatility and Sharpe ratio of each portfolio are denoted by $\mu_p$, $\sigma_p$ and $SR$ respectively. The dynamic strategies use the maximum utility, maximum return or minimum volatility rule to build a portfolio by investing in the daily return of six bonds from the US, Canada, UK, Germany, Switzerland and Japan and using the five exchange rate forecasts to convert the portfolio return in US dollars. The performance fees ($\Phi$) denote the amount an investor with quadratic utility and a degree of relative risk aversion equal to 6 is willing to pay for switching from MLR to one of the dynamic models. The performance fees are expressed in annual basis points and are reported for three levels of proportional transaction costs: 0, 1 bps and 2 bps. $M2$ is the Modigliani and Modigliani (1997) measure of the abnormal return a dynamic strategy would have earned if it had the same risk as the static benchmark and is measured annualized basis points. The break-even transaction cost $\tau^{BE}$ is defined as the minimum daily proportional cost which cancels out the utility advantage (and hence positive performance fee) of a given strategy. The $\tau^{BE}$ values are expressed in daily basis points.
References


