A component model for dynamic correlations

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Abstract

The idea of component models for volatility is extended to dynamic correlations. We propose a model of dynamic correlations with a short- and long-run component specification. We call it the class of models DCC-MIDAS as the key ingredients are a combination of the Engle (2002) DCC model, the Engle and Lee (1999) component GARCH model to replace the original DCC dynamics with a component specification and the Engle, Ghysels, and Sohn (2006) GARCH-MIDAS component specification that allows us to extract a long-run correlation component via mixed data sampling. We provide a comprehensive econometric analysis of the new class of models, including conditions for positive semi-definiteness, and provide extensive empirical evidence that supports the model specification.

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1 Introduction

Component models have been widely used for volatility dynamics. The motivation is typically based on either one of the following two arguments. First, the component structure allows for a parsimonious representation of complex dependence structures. Second, the components are sometimes linked to economic principles, namely the idea that there are different short- and long-run sources that affect volatility. The purpose of this paper is to propose a component model of dynamic correlations with a short- and long-run component specification.\(^1\) We call it the class of models DCC-MIDAS as the key ingredients are a combination of the Engle (2002) DCC model, the Engle and Lee (1999) component GARCH model to replace the original DCC dynamics with a component specification and the Engle, Ghysels, and Sohn (2006) GARCH-MIDAS component specification that allows us to extract a long-run correlation component via mixed data sampling.

We address the specification, estimation and interpretation of correlation models that distinguish short and long run components. We show that the changes in correlations are indeed very different. Dynamic correlations are a natural extension of the GARCH-MIDAS model to Engle (2002) DCC model. The idea captured by the DCC-MIDAS model is similar to that underlying GARCH-MIDAS. In the latter case, two components of volatility are extracted, one pertaining to short term fluctuations, the other pertaining to a secular component. In the GARCH-MIDAS the short run component is a GARCH component, based on daily (squared) returns, that moves around a long-run component driven by realized volatilities computed over a monthly, quarterly or bi-annual basis. The MIDAS weighting scheme helps us extracting the slowly moving secular component around which daily volatility moves. Engle, Ghysels, and Sohn (2006) explicitly link the extracted MIDAS component to macroeconomic sources. It is the same logic that is applied here to correlations. Namely, the daily dynamics obey a DCC scheme, with the correlations moving around a long run component. Short-lived effects to correlations will be captured by the autoregressive dynamic structure of DCC, with the intercept of the latter being a slowly moving process that reflects the fundamental or long-run causes of time variation in correlation.\(^2\)

\(^1\)It should be noted that there have been several prior attempts to think of component models for correlations, see inter alia Karolyi and Stulz (1996). Our approach focuses on autoregressive conditional correlation models.

\(^2\)In principle we can link the secular correlation component to macroeconomic sources, very much like Engle, Ghysels, and Sohn (2006) study long historical time series, similar to Schwert (1989) and link volatility directly to various key macroeconomic time series.
To estimate the parameters of the DCC-MIDAS model we follow the two-step procedure of Engle (2002). We start by estimating the parameters of the univariate conditional volatility models. The second step consists of estimating the DCC-MIDAS parameters with the standardized residuals. We also discuss the regularity conditions we need to impose on the MIDAS-filtered long run correlation component as models of correlations are required to yield positive definite matrices.

The paper concludes with an empirical illustration, showing the benefits of the component specification. Empirical specification tests are introduced and applied. They reveal the superior empirical fit, both in- and out-of-sample of the new class of DCC-MIDAS correlation models.

The remainder of the paper is organized as follows. Section 2 introduces the correlation component model and compares the DCC-MIDAS class of models with original DCC models. Section 3 covers regularity conditions and estimation, while section 5 contains the empirical applications. Section 6 concludes the paper.

## 2 A new class of component correlation models

Consider a set of $n$ assets and let the vector of returns be denoted as $\mathbf{r}_t = [r_{1,t}, \ldots, r_{n,t}]'$. The novelty of our approach consists of describing the dynamics of conditional variances and correlations, where we take into account both short and long run components. The long run component at time $t$ will be a judiciously chosen weighted average of historical correlations. The assumption is that the long run component can be filtered from empirical correlations. Of course, what is critical is the choice of weights, which will be one of the key ingredients of the model specification. To proceed let us assume that the vector of returns $\mathbf{r}_t = [r_{1,t}, \ldots, r_{n,t}]'$ follows the process:

$$
\begin{align*}
\mathbf{r}_t & \sim_{i.i.d.} N(\mathbf{\mu}, H_t) \\
H_t &= D_t R_t D_t
\end{align*}
$$

(2.1)
\( \mu \) is where the vector of unconditional means, \( H_t \) is the conditional covariance matrix and \( D_t \) is a diagonal matrix with standard deviations on the diagonal, and:

\[
R_t = E_{t-1}[\xi_t \xi_t'] \quad (2.2)
\]

\[
\xi_t = D_t^{-1}(r_t - \mu)
\]

Therefore \( r_t = \mu + H_t^{1/2} \xi_t \) with \( \xi_t \sim_{i.i.d.} N(0, I_n) \). In this section we introduce a new class of component correlation models. In a first subsection we discuss model specification. The next subsection covers regularity conditions and deals with estimation.

At the outset, it should be noted that component models for correlations also prompt us to think about component models for volatility which feed into the correlation specification. Indeed the decomposition of the conditional covariance matrix \( H_t = D_t R_t D_t \) appearing in equation (2.1) with \( D_t \) a diagonal matrix of standard deviations and \( R_t \) the conditional correlation matrix suggests a two-step model specification (and estimation) strategy. Consequently, we will first specify \( D_t \) followed by \( R_t \).

The univariate volatility models build on recent work by Engle and Rangel (2005) and in particular Engle, Ghysels, and Sohn (2006). Both proposed component models for volatility, where long an short run volatility dynamics are separated. Engle and Rangel (2005) introduce a Spline-GARCH model where the daily equity volatility is a product of a slowly varying deterministic component and a mean reverting unit GARCH. Unlike conventional GARCH or stochastic volatility models, this model permits low frequency volatility to change over time. Engle and Rangel (2005) use an exponential spline as a convenient non-negative parameterization. The recent work of Engle, Ghysels, and Sohn (2006) combines insights from Engle and Rangel (2005) with a framework that is suited to combine data that are sampled at different frequencies. The new approach is inspired by the recent work on mixed data sampling, or MIDAS discussed in Ghysels, Santa-Clara, and Valkanov (2005), Ghysels, Santa-Clara, and Valkanov (2006), Forsberg and Ghysels (2004), among others.\(^3\) Engle, Ghysels, and Sohn (2006) replace the spline specification with a MIDAS polynomial. The new class of models is called GARCH-MIDAS, since it uses a mean reverting unit daily GARCH

\(^3\)In the context of volatility, Ghysels, Santa-Clara, and Valkanov (2005) studied the traditional risk-return trade-off and used monthly data to proxy expected returns while the variance was estimated using daily squared returns. The idea was carried a step further in Ghysels, Santa-Clara, and Valkanov (2006) and Forsberg and Ghysels (2004), both papers focusing on predicting volatility at various horizons with high frequency financial data using MIDAS.
process, similar to Engle and Rangel (2005), and a MIDAS polynomial which applies to monthly, quarterly, or bi-annual macroeconomic or financial variables. More specifically we assume that univariate returns follow the GARCH-MIDAS process:

\[ r_{i,t} = \mu_i + \sqrt{\tau_{i,t}} g_{i,t} \xi_{i,t}, \quad \forall i = 1, \ldots, n \]  

(2.3)

where for each \( i \), \( g_{i,t} \) follows a GARCH(1,1) process:

\[ g_{i,t} = (1 - \alpha_i - \beta_i) + \alpha_i \frac{(r_{i,t} - \mu_i)^2}{\tau_{i,t}} + \beta_i g_{i,t-1} \]  

(2.4)

while the component \( \tau_{i,t} \) is:

\[
\begin{align*}
\tau_{i,t} &= m_i + \theta_i \sum_{l=1}^{L_i} \varphi_l(\omega^i_v) RV_{i,t-l} \\
RV_{i,t} &= \sum_{j=1}^{N_i^v} (r_{i,t-j})^2 \\
\varphi_l(\omega^i_v) &= \left(1 - \frac{l}{L_i^v}\right)^{\omega^i_v-1} \\
&\quad \frac{\sum_{j=1}^{L_i^v} \left(1 - \frac{j}{L_i^v}\right)^{\omega^i_v-1}}
\end{align*}
\]  

(2.5, 2.6)

In what follows we will refer to \( g_{i,t} \) and \( \tau_{i,t} \) as the short and long run variances respectively. Note that the long run component is essentially a weighted sum of \( L_i^v \) lags of realized volatilities over a long horizon, where the realized volatility involve daily squared returns over a span of length \( N_i^v \) which may vary with each series. In practice we will consider cases where the parameters \( N_i^v \) and \( L_i^v \) are independent of \( i \), i.e. the same across all series. Similarly, we can also allow for different decay patterns \( \omega_i \) across various series, but once again we will focus on cases with common \( \omega \) (see the next subsection for further discussion). Obviously, despite the common parameter specification, we expect that the \( \tau_{i,t} \) substantially differ across series, as they are data-driven.

Dynamic correlations are a natural extension of the GARCH-MIDAS model to Engle (2002) DCC model. Using the standardized residuals it is possible to obtain a matrix \( Q_t \) whose
where the weighting scheme is similar to that appearing in (2.5). Correlations can then be computed as:

$$\rho_{i,j,t} = \frac{q_{i,j,t}}{\sqrt{q_{i,i,t}q_{j,j,t}}}$$

We regard $q_{i,j,t}$ as the short run correlation between assets $i$ and $j$, whereas $\bar{\rho}_{i,j,t}$ is a slowly moving long run correlation. Rewriting the first equation of system (2.7) as

$$q_{i,j,t} - \bar{\rho}_{i,j,t} = a\left(\xi_{i,t-1}\xi_{j,t-1} - \bar{\rho}_{i,j,t}\right) + b\left(q_{i,j,t-1} - \bar{\rho}_{i,j,t}\right)$$

(2.8)

conveys the idea of short run fluctuations around a time varying long run relationship. The idea captured by the DCC-MIDAS model is similar to that underlying GARCH-MIDAS. In the latter case, two components of volatility are extracted, one pertaining to short term fluctuations, the other pertaining to a secular component. In the GARCH-MIDAS the short run component is a GARCH component, based on daily (squared) returns, that moves around a long-run component driven by realized volatilities computed over a monthly, quarterly or bi-annual basis. The MIDAS weighting scheme helps us extracting the slowly moving secular component around which daily volatility moves. Engle, Ghysels, and Sohn (2006) explicitly link the extracted MIDAS component to macroeconomic sources. It is the same logic that is applied here to correlations. Namely, the daily dynamics obey a DCC scheme, with the correlations moving around a long run component. Short-lived effects on correlations will be captured by the autoregressive dynamic structure of DCC, with the intercept of the latter being a slowly moving process that reflects the fundamental or secular causes of time variation in correlation. In principle we can link the long run correlation component to macroeconomic sources, very much like Engle, Ghysels, and Sohn (2006) study long historical time series, similar to Schwert (1989) and link volatility directly to various key macroeconomic time
series.\(^4\) Note that in equation (2.5) we can allow for different weighting schemes across series. Likewise, the specification in (2.7) can potentially accommodate weights \(\omega^{ij}_r\), lag lengths \(N_{ij}^c\) and span lengths of historical correlations \(L_{ij}^c\) to differ across any pair of series.\(^5\)

Typically we will use a single setting common to all pairs of series, similar to the choice of a common MIDAS filter in the in the univariate models. We will discuss in the next subsection the implications of a single versus multiple parameter choices for the DCC-MIDAS filtering scheme.

It is also worth noting that our DCC-MIDAS model shares features with a local dynamic conditional correlation (LDCC) model introduced in Feng (2007), where variances are decomposed into a conditional and a local (unconditional) parts. The correlation structure is modeled by a multivariate nonparametric ARCH-type approach that accommodates the presence of regressors.

To conclude this subsection we fix some notation that will allow us to discuss the general model specification. First, we will collect all the elements \(\omega^{ij}_r\) into a vector \(\omega_r\), keeping in mind that it may only contain a single element if all weights are equal and we denote \(N_c = \max_{i,j} N_{ij}^c\). We then can write \(R_t = (Q_t^*)^{-1/2}Q_t(Q_t^*)^{-1/2}\) and \(Q_t^* = \text{diag}Q_t\), and collect the set of correlations appearing in equation (2.7) yielding generically in matrix form:

\[
Q_t = (1-a-b)\overline{R}_t(\omega_r) + a\xi_t\xi_t' + bQ_{t-1}
\]

\[
\overline{R}_t(\omega_r) = \sum_{l=1}^{L_c} \varphi_l(\omega_r) C_{t-l}
\]

\[
C_t = \begin{pmatrix}
 v_{1,t} & 0 & 0 \\
 \vdots & \ddots & 0 \\
 0 & \cdots & v_{n,t}
\end{pmatrix}^{-\frac{1}{2}} \left( \sum_{k=t-N_c}^{t} \xi_k\xi_k' \right) \begin{pmatrix}
 v_{1,t} & 0 & 0 \\
 \vdots & \ddots & 0 \\
 0 & \cdots & v_{n,t}
\end{pmatrix}^{-\frac{1}{2}}
\]

\[
v_{i,t} = \sum_{k=t-N_c}^{t} \xi^2_{i,k}, \quad \forall i = 1, \ldots, n
\]

\(^4\)We prefer not to do this in the current paper, because it would require selecting an ad hoc function to link macro variables to correlations. This function should satisfy the restrictions that correlations are bounded between \(-1\) and \(1\) and the resulting correlation matrix must be positive semi-definite.

\(^5\)Note that \((\omega_r^{ij}, N_c^{ij}, L_c^{ij}) = (\omega_r^{ji}, N_c^{ji}, L_c^{ji})\) are identical for all \(i\) and \(j\).
2.1 A preliminary simulation study

It is hard to say a priori whether the new class of models we propose has any potential at improving say forecasting correlations. To make any assessment of this we need to know the true correlation structure and see how DCC-MIDAS compares to the original DCC model. Fortunately, there is a reasonable setting to make such a comparison. In the original paper Engle (2002) considered a number of correlation dynamics and evaluated how the original DCC model tracked the different assumed correlation dynamics. We revisit the same set of correlation patterns and compare the DCC with the new class of DCC-MIDAS.

We consider the simplest case of a two by two system and we assume that variances follow a GARCH-MIDAS model with parameters set equal to those estimated for the variance for one of the series that we will consider later in the empirical work, namely the Energy industry in Table 2. Correlations are assumed to follow one of the following seven processes:

1. Step: \( \rho_t = \begin{cases} 0.4, & \forall t < 500 \\ 0.9, & \forall t \geq 500 \end{cases} \)

2. Large Step: \( \rho_t = \begin{cases} -0.5, & \forall t < 500 \\ 0.5, & \forall t \geq 500 \end{cases} \)

3. Double Step: \( \rho_t = \begin{cases} -1/3, & \forall t < 334 \\ 2/3, & 334 \leq t < 666 \\ -1/3, & \forall t \geq 667 \end{cases} \)

4. Long Sample Double Step: \( \rho_t = \begin{cases} -1/3, & \forall t < 500 \\ 2/3, & 500 \leq t < 1000 \\ -1/3, & \forall t \geq 1000 \end{cases} \)

5. Ramp: \( \rho_t = \text{mod}(t/200) \)

6. Short Cycle: \( \rho_t = \sin(2\pi t/500) \)

7. Long Cycle: \( \rho_t = \sin(2\pi t/1000) \)

For each correlation pattern 100 samples of size 1000 were simulated, with the only exception of case 4 for which the sample size was 1500. Figure 1 reports the patterns of the simulated correlations. A priori the only scenarios in which we would expect the DCC-MIDAS...
to behave better than the original DCC are those of the step functions. By construction
the DCC-MIDAS filters out a time varying long run correlation around which the short run
dynamics takes place. The original DCC would treat the long-run unconditional correlation
as time invariant and greatly exaggerate the short run dynamics to fit the proposed correlation pattern. This is indeed what we find in Table 1. As expected the DCC-MIDAS seems
to provide a significant improvement over the basic DCC already in the case of the step
function of size 0.5. The result is even more apparent when the size of the step is increased
to 1. In the event of a double step over a sample of size 1000, the DCC-MIDAS does not
improve dramatically the fit over the original DCC. This is probably due to the fact that
the different regimes are short-lived spanning only 333 days each. The table shows that as
the sample size of each regime was increased to 500, the performance of the DCC-MIDAS
seemed to improve over the original DCC. In all other cases the differences are minimal. The
latter result is also not too surprising as in none of these cases the patterns of the simulated
correlations are affected by a shift in the long term dynamics of some kind. In a sense, we
can also look upon this as an example of regime switching DCC models, see e.g. Pelletier
(2006). Regime-switching DCC models may be also thought of has capturing changes in the
overall level of correlation (albeit with revisiting regularly the same regime through time).

— Insert Table 1 about here —

3 Estimation

To estimate the parameters of the DCC-MIDAS model we follow the two-step procedure of
Engle (2002). We start by collecting the parameters of the univariate conditional volatility
models into a vector \( \Psi \equiv [(\alpha_i, \beta_i, \omega_i, m_i, \theta_i), i = 1, \ldots, n] \), and the parameters of the conditional correlation model into \( \Xi \equiv (a, b, \omega_r) \). Then the (quasi-)likelihood function \( QL \) can be
written as:

\[
QL(\Psi, \Xi) = QL_1(\Psi) + QL_2(\Psi, \Xi) = -\sum_{t=1}^{T} \left( n \log(2\pi) + 2 \log |D_t| + r_t' D_t^{-2} r_t \right) - \sum_{t=1}^{T} \left( \log |R_t| + \xi_t' R_t^{-1} \xi_t + \xi_t' \xi_t \right)
\]

Given the structure of the log likelihood function, namely the separation of \( QL(\Psi, \Xi) \) into
\( QL_1(\Psi) \) and \( QL_2(\Psi, \Xi) \), we can first estimate the parameters of the univariate GARCH-
MIDAS processes, i.e. the parameters in $\Psi$, using $QL_1(\Psi)$ and therefore each single series separately - yielding $\hat{\Psi}$. The second step consists of estimating the DCC-MIDAS parameters with the standardized residuals $\hat{\xi}_t = \hat{D}_t^{-1}(r_t - \hat{\mu})$ using $QL_2(\hat{\Psi}, \Xi)$. The estimation of the MIDAS polynomial parameters in the dynamic correlations require some further discussion. The approach we adopt is inspired by the estimation of MIDAS polynomial parameters in the GARCH-MIDAS model.

So far we were not very explicit about the choice of the polynomial characteristics $L^i_v$ and $N^i_v$ in equation (2.5) and the choice of $L_c$ and $N_c$ in equation (2.7). In the former case, i.e. the univariate GARCH-MIDAS models, $L^i_v$ determines the number of lags spanned in each MIDAS polynomial specifications for $\tau_t$. The other is how to compute RV, weighted by the MIDAS polynomials. As pointed out by Engle, Ghysels, and Sohn (2006), this amounts to model selection with a fixed parameter space, and therefore is achieved via profiling the likelihood function for various combinations of $L^i_v$ and $N^i_v$. To determine the long run component of conditional correlations, $\overline{R}_t$ we proceed in exactly the same way, namely we select the number of lags $L_c$ for historical correlations and the time span over which to compute the historical correlations $N_c$ in equation (2.7). The similarity between the two procedures is not surprising, given the fact that DCC models build extensively on the ideas of GARCH and in both cases we have a MIDAS filter extracting a component which behaves like a time-varying intercept. We will provide an explicit discussion of the procedure in the empirical applications, given the similarity with Engle, Ghysels, and Sohn (2006).

4 Regularity Conditions

The asymptotic properties of the two-step estimator are discussed in Engle and Sheppard (2001), Comte and Lieberman (2003), Ling and McAleer (2003) and McAleer, Chan, Hoti, and Lieberman (2006). These papers deal with fixed parameter DCC models. It is beyond the scope of the current paper to establish the asymptotic properties of the MLE estimator when the MIDAS random intercept is present. Recent work by Dahlhaus and Subba Rao (2006) discussed general time-varying coefficient ARCH($\infty$) processes and regularity conditions for (local) MLE estimation. The GARCH-MIDAS and DCC-MIDAS processes are to a certain degree special cases of their setup. Namely, Dahlhaus and Subba Rao (2006) allow all parameters to vary and assume a nonparametric setting to capture the time-varying coef-
ficients. This leads them, like Feng (2007), to consider kernel-based estimators. Our setting is parametric, as the MIDAS filter is a parametric specification, and therefore presumably simpler. We leave the regularity conditions that guarantee standard asymptotic results for the two-step estimation of DCC-MIDAS as an open question for future research. However, we do cover in this paper the regularity conditions we need to impose on the MIDAS-filtered long run correlation component to obtain positive definite matrices. More specifically, we turn our attention to the long run component and the choice of weights $\omega_{ij}$, keeping the lag lengths $N_{ij}^c$ and span lengths of historical correlations $L_{ij}^c$ fixed across all pairs of series. Hence, we focus on the memory decay in the long run correlations.

4.1 Long-Run dynamics

The first case to consider is the one of a common decay parameter $\omega_r$ independent of the pair of returns series selected. The covariance matrices can be shown to be positive definite under a relatively mild set of assumptions. When considering equation (2.9) it is apparent that the matrix $Q_t$ is a weighted average of three matrices. The matrix $R_t$ is a positive semi-definite because it is a weighted average of correlation matrices. The matrix $\xi_t \xi_t'$ is always positive semi-definite by construction. Therefore, if the matrix $Q_0$ is initialized to a positive semi-definite matrix, it follows that $Q_t$ must be positive semi-definite at each point in time.

The positive semi-definiteness of the covariance matrix can be guaranteed without putting any restriction on the structure of the conditional variance estimators for the individual return series. This means, for example, that it is possible to assume a different number of GARCH-MIDAS lags for each return.

The case of two weighting schemes is more involved and therefore becomes more interesting. To discuss it, we shall adopt the following notation:

$$R_t(\omega_r) = \begin{bmatrix} R_{\text{pp}}^t(\omega_r) & R_{\text{pN}}^t(\omega_r) \\ R_{\text{Np}}^t(\omega_r)' & R_{\text{NN}}^t(\omega_r) \end{bmatrix}$$

where $R_{\text{pp}}^t(\omega_r)$ is the first square matrix of size $p$, $R_{\text{NN}}^t(\omega_r)$ is the last square matrix of size $N - p$, and $R_{\text{pN}}^t(\omega_r)$ is the off-diagonal $p$ by $N - p$ matrix. In the case that two MIDAS
filters are being used, we shall decompose the long-run correlation matrix as follows:

\[
\mathbf{R}_t(\omega_r) = \begin{bmatrix}
\mathbf{R}^{pp}_t(\omega^a_r) & \mathbf{R}^{pN}_t(\omega^a_r) \\
\mathbf{R}^{pN}_t(\omega^a_r)' & \mathbf{R}^{NN}_t(\omega^b_r)
\end{bmatrix}
\]

where \(\omega_r = (\omega^a_r, \omega^b_r)\). Hence, the Beta polynomial with parameter \(\omega^b_r\) is applied to the correlations of the last \(N - p\) assets, while the Beta polynomial with parameter \(\omega^a_r\) describes the dynamics of any other correlation. Positive semi-definiteness of \(\mathbf{R}_t(\omega_r)\) is not automatically guaranteed. However, we can always ensure that a proper correlation matrix exists provided that the following assumption is satisfied:

**Assumption 1.** In each sample of size \(L_c\) there exists a correlation matrix \(\mathbf{C}_t\) that is positive definite.

This assumption is very mild. In practice it amounts to requiring that in each sample that is used to construct the long-run correlation matrix there is at least one point in time in which two series are not perfectly correlated. In Appendix A we prove the following:

**Proposition 1** (Existence of a correlation matrix). If Assumption 1 is satisfied, then the set \(\{\omega^b_r : \omega^b_r \neq \omega^a_r \land \mathbf{R}_t(\omega_r)\text{ is positive definite}, \forall t\}\) is non-empty and non-singleton.

It is worth illustrating the result with the simplest case of the generalization to multiple filters: two MIDAS correlation polynomials and a 3 by 3 system. Let

\[
\mathbf{R} = \begin{bmatrix}
1 & r_{12,t} & r_{13,t} \\
r_{12,t} & 1 & r_{23,t} \\
r_{13,t} & r_{23,t} & 1
\end{bmatrix}
\]

be the long run correlation matrix that would result from the application of a common MIDAS filter

\[
r_{ij,t} = \sum_{l=1}^{L_c} \varphi_l(\omega^a_r) r_{ij,t-l}
\]

As we argued before, since this matrix is a weighted average of positive definite matrices it will be positive definite or equivalently: \(\det(\mathbf{R}) > 0\).
We want to study the sign of the minors of the matrix
\[
\tilde{R} = \begin{bmatrix}
1 & r_{12,t} & r_{13,t} \\
1 & r_{12,t} & r_{23,t} + \lambda_t \\
r_{13,t} & r_{23,t} + \lambda_t & 1
\end{bmatrix}
\]
where
\[
r_{23,t} + \lambda_t = \sum_{l=1}^{L_c} \varphi_l (\omega^b_r) \rho_{23,t-l}
\]
for some \( \omega^b_r \) different from \( \omega^a_r \). The first two minors are always non-negative by construction. The third minor can be written as:
\[-\lambda_t^2 - 2 (r_{23,t} - r_{12,t} r_{13,t}) \lambda_t + \det(R)\]
For it to be positive, it must be the case that
\[\lambda_t \leq \lambda_t \leq \tilde{\lambda}_t\] (4.2)
where
\[
\lambda_t = r_{12,t} r_{13,t} - r_{23,t} - \sqrt{(r_{12,t} r_{13,t} - r_{23,t})^2 + \det(R)}
\]
\[
\tilde{\lambda}_t = r_{12,t} r_{13,t} - r_{23,t} + \sqrt{(r_{12,t} r_{13,t} - r_{23,t})^2 + \det(R)}
\]
Note that \( \lambda_t \) is always negative and \( \tilde{\lambda}_t \) is always positive, because \( \det(R) > 0 \). For a given \( \omega^a_r \), condition (4.2) restricts \( \omega^b_r \) by an amount that is a direct function of \( \det(R) \).

In many practical applications we may want to use even more general specifications for the correlation matrix. For instance the joint dynamics of a subset of the series may be described by the MIDAS parameter \( \omega^a_r \), another by \( \omega^b_r \), and all the cross-correlations between the two subsets by \( \varphi^c_r \). The following block matrix representation formalizes the example:
\[
\overline{R}_t(\omega_r) = \begin{bmatrix}
\overline{R}_t^{pp}(\omega^a_r) & \overline{R}_t^{pN}(\omega^c_r) \\
\overline{R}_t^{Np}(\omega^c_r)' & \overline{R}_t^{NN}(\omega^b_r)
\end{bmatrix}
\]
with \( \omega_r = \{ \omega^a_r, \omega^b_r, \omega^c_r \} \). Also in this case it is possible to ensure the existence of a set \( \omega_r \) for which \( \overline{R}_t(\omega_r) \) is a positive semi-definite matrix. This follows as a corollary of proposition 1:
Corollary 1. If Assumption 1 is satisfied, then the set
\[
\left\{ \omega^i_r : \omega^i_r \neq \omega^j_r \right\}_{i \neq j} \cap \left\{ R_t (\omega^a_r, \omega^b_r, \omega^c_r) \right\} \text{ is positive definite, } \forall t \}
\]
is non-empty and non-singleton.

Proof. See Appendix A.

4.2 Short-Run dynamics

In general it will also prove convenient to allow for multiple sets of parameters to describe the DCC part of the correlation dynamics. Along the lines of the above examples concerning long run correlations, we may want the parameters \((a_1, b_1)\) accounting for the short run correlations between assets 1 and 2, the parameters \((a_2, b_2)\) accounting for the short run correlations between assets 3 and 4, and the parameters \((a_3, b_3)\) for the cross correlations dynamics. In formulas, let \(N\) be the size of the matrix \(Q^d_t\), let \(p\) and \(N-p\) be the sizes of the block diagonal matrices that share the parameters \((a_2, b_2)\) and \((a_3, b_3)\) respectively, and let \((a_1, b_1)\) account for the cross-dynamics. Then the version of matrix \(Q_t\) in (2.9) that applies is

\[
Q^d_t = \sum_{j=1}^{3} G_j \left[ \tilde{c}_j \tilde{R}_t(\omega_r) + \tilde{a}_j \xi_t \xi_t' + \tilde{b}_j Q^d_{t-1} \right] G_j
\]

(4.3)

where

\[
\tilde{a}_1 = a_1, \quad \tilde{b}_1 = b_1, \quad \tilde{c}_1 = 1 - \tilde{a}_1 - \tilde{b}_1
\]

\[
\tilde{a}_j = a_j - a_1, \quad \tilde{b}_j = b_j - b_1, \quad \tilde{c}_j = -\tilde{a}_j - \tilde{b}_j, \quad \forall j \geq 2
\]

and

\[
G_1 = I_n, \quad G_2 = \begin{bmatrix} I_p & 0_{p, (N-p)} \\ 0_{(N-p), p} & 0_{(N-p), (N-p)} \end{bmatrix}, \quad G_3 = \begin{bmatrix} 0_{p, p} & 0_{p, (N-p)} \\ 0_{(N-p), p} & I_{(N-p)} \end{bmatrix}
\]

Under an assumption about the positive definiteness of the sequence of matrices \(\xi_t \xi'_t\) that
resembles assumption 1 and suitable initial conditions, it is possible to state a theorem concerning the existence of sequences of positive semi-definite $Q^d_t$.

**Assumption 2.** The matrix $\xi_t \xi_t'$ is positive definite $\forall t$.

This assumption will be satisfied in any sample in which no two series are perfectly correlated at any point in time. It will also imply assumption 1 for the case in which the sequence of correlation matrices used to filter out the secular component is a function only of lagged correlations. The following proposition can be stated.

**Proposition 2 (Existence of a positive definite $Q^d_t$).** If assumption 2 is satisfied and $Q^d_0$ is positive semi-definite, then there exist sets of parameters $\{a_1, a_2, a_3\}$ and $\{b_1, b_2, b_3\}$ with elements not all equal for which $Q^d_t$ is positive semi-definite $\forall t$.

We will skip the proof of Proposition 2, because it mimics that of Proposition 1 and its corollary. In the remainder of the paper we devote our attention to empirical examples as well as comparison of the DCC-MIDAS class of correlation models with the original DCC specifications.

### 4.3 An alternative specification

The specifications described in the previous subsections have the nice feature of being straightforward generalizations of the case in which a single pair of parameters is applied to the short-run dynamics and a single MIDAS filter describes long-run correlations. The only drawback is that the statement about the positive semi-definiteness of the covariance matrices is not sample independent. As long as the analysis is focused on in-sample forecasts this is not an issue, but it may be desirable to work with a model for which covariance matrices are always PSD.

Let there be $J$ pairs of DCC parameters $\{a_j, b_j\}_{j=1}^J$ and $J$ MIDAS filters $\{\phi^j(\omega_r)\}_{j=1}^J$. The following specification is an extension of the generalized DCC model of Cappiello, Engle,
and Sheppard (2003):

\[ Q_t^g = (\omega' - aa' - bb')R_t(\omega_r) + aa' \odot \xi_{t-1} \xi_{t-1}' + bb' \odot Q_{t-1}^g \quad (4.4) \]

\[ R_t(\omega_r) = \sum_{l=1}^{L_c} \varphi_l(\omega, \omega)' \odot C_{t-l} \]

where

\[ a = \left[ \begin{array}{cccc} \sqrt{a_1} & \ldots & \sqrt{a_2} & \ldots & \sqrt{a_J} \end{array} \right] \]

\[ b = \left[ \begin{array}{cccc} \sqrt{b_1} & \ldots & \sqrt{b_2} & \ldots & \sqrt{b_J} \end{array} \right] \]

\[ \varphi_l(\omega, \omega) = \left[ \begin{array}{cccc} \sqrt{\varphi_{l1}} & \ldots & \sqrt{\varphi_{l2}} & \ldots & \sqrt{\varphi_{lJ}} \end{array} \right] \]

The following proposition guarantees that under very mild assumptions, the matrix \( Q_t^g \) is always positive semi-definite.

**Proposition 3 (Generalized DCC-MIDAS).** If \( (\omega' - aa' - bb') > 0 \), then the matrix \( Q_t^g \) is always positive semi-definite.

**Proof.** See Appendix A.

In the empirical applications, we show that for systems of small dimensions, using the Generalized DCC-MIDAS or the specifications reported in the previous sub-sections does not make any significant difference.

## 5 Empirical Applications

We study various empirical examples, starting with a bi-variate system, next moving to two examples with three assets. For the equities, the portfolios are formed based on an industry

---

6The symbol \( \odot \) stands for the Hadamard product.
classification. In the bi-variate case, we only have one MIDAS filter, hence the discussion pertaining to regularity conditions for positive semi-definiteness in the previous section does not apply. We therefore move to the tri-variate systems to discuss the selection of MIDAS filters. Each of the examples are designed to highlight model estimation and specification issues. The section is divided in subsections which cover the various examples.

5.1 An example of two assets: Energy portfolio and 10 year bond

We start the investigation of short and long run correlation dynamics of industries portfolios and 10 year bond, by focusing on a bi-variate case that involves the energy industry portfolio and the 10 year bond only. It is convenient to outline the various steps involved in the estimation procedure in the context of this simple case.

We consider a sample of daily returns on an industry portfolio - the energy sector - and a long term bond, namely a 10-year bond. The sample starts on 1971-07-15 and ends on 2006-06-30. We first address the issue of selecting the number of MIDAS lags. We follow a procedure suggested in Engle, Ghysels, and Sohn (2006), since the GARCH-MIDAS and DCC-MIDAS class of models share similar model selection issues. A convenient property of both models, is that the lag selection of the MIDAS filter involves a fixed number of parameters. Hence, Engle, Ghysels, and Sohn (2006) compare various GARCH-MIDAS models with different time spans via profiling of the likelihood function. The task can be easily accomplished by looking at the plots of the log-likelihood functions of the GARCH-MIDAS and DCC-MIDAS estimators for an increasing number of lags. The left panel of Figure 2 shows that a small number of MIDAS lags is typically enough to accurately describe the long run dynamics of the volatilities of the two series. We select the smallest number of MIDAS lags after which the log-likelihoods of the two volatilities seem to reach their plateau. In this case this criterion amounts to picking 36 lags of monthly realized volatilities. The right panel of Figure 2 shows that DCC-MIDAS requires a larger number of lags before its log-likelihood flattens out. This led us to select 144 lags.

--- Insert Figure 2 about here ---

Figure 3 reports the short and long run dynamics of volatilities and correlations. The dark lines in the figure correspond to the long run correlation with short run correlations snaking
around it. These results show that despite the fact that the unconditional correlation of the two series is close to zero, the short-run correlation is characterized by large departures from this value. The long-run correlation slowly adjusts to account for periods of relatively higher and relatively lower correlation. Table 2 reports the estimates of the parameters of the DCC-MIDAS and of the original DCC, where the latter is modified to take into account the long-run dynamics of volatilities only. It is interesting to note that the persistence parameters $\beta$ of the two GARCH processes are remarkably lower than we typically observe when neglecting the long-run dynamics for daily series. The same is true also for the persistence parameter of the correlation $b$. Both facts can be attributed to the slowly moving MIDAS adjustment.

Figure 4 shows how much the short run correlations are affected by ignoring time-varying long-run dynamics. The solid line represents the differences in correlations between the original DCC and the DCC-MIDAS. These differences can be as high as 0.05 in absolute value and the picture shows that they are typically positive when the time varying long run correlation exceeds the unconditional one and negative when the relationship between the two correlations is reversed.

### 5.2 More than two assets

When we consider more than two assets we have the possibility that several long run MIDAS filters apply. We provide two examples involving three asset returns, one where a single MIDAS filter suffices, and another where there is clearly a need for two filters. The former involves two industries and a bond, namely Energy and Hi-Tech portfolios vs. 10 year bond. The results appear in Figures 5 and 6 as well as Table 3. The first figure displays Energy-HiTech-10 year Bond Variances and correlation and the second shows the differences with DCC and unconditional correlations.
In Table 6 we report likelihood ratio tests for various nested model specifications involving separate parameters for the DCC dynamics and/or MIDAS filters. Each entry in the table represents the p-value for testing that the likelihood of the model of the column is significantly higher than the likelihood of the model on the corresponding row. We find that none of the tests are significant at conventional confidence levels. Overall the findings are similar to two asset example in the previous subsection and the results show that the two industries have the same dynamic patterns for long run correlation and how they relate to fixed income.

The next and final example shows that this is not always the case. In Table 7 we report likelihood ratio tests for Ten year Bonds combined with Manufacturing and Shops industries. The parameter estimates, for the single MIDAS filter appear in Table 4 whereas the multiple filter case appears in Table 5. The bottom part of the table shows that when the MIDAS parameters are estimated in all possible permutations of bivariate systems, a value close to 7 appears to do the job for bond vs. manufacturing and for for bond vs. shops. A decisively shorter memory achieves the maximum likelihood when it comes to accounting for the long run dynamics of the correlation between shops and manufacturing. The top part of Table 5 shows that it can indeed be quite restrictive to force one MIDAS parameter to describe the long-run dynamics of all pairs of correlations. The introduction of an additional MIDAS parameter not only brings the outcome of the estimation closer to what suggested by the analysis of the bivariate systems, but it also sizeably increases the log-likelihood. Table 7 shows that this increase is significant at a 1% confidence level.

The variances and correlations appear respectively in Figures 7 and 9. The former shows the single filter patterns and the latter shows the patterns with two distinct filters. We observe that the second filter clearly changes the long run component correlation across the two industries. Figure 6 as well as figure 8 confirm once again that the original DCC
seems to overshoot on average the short run correlation when the long run one is above its unconditional value, while the opposite happens when the MIDAS long run correlation lies below its unconditional counterpart.

When we employ the Generalized DCC-MIDAS model, we obtain the parameters’ estimates reported in table 5. These estimates seem to confirm the need for a second MIDAS filter to be applied to the correlation between the manufacturing and the shops portfolios. Figure 10 shows that the long-run correlations filtered using this specification appear to be a little smoother when the 10-year bond is one of the assets compared to the results obtained under the previous specification. The low-frequency correlation of the two portfolios is instead a little noisier. Aside for these small differences, we take the results as confirming the need for multiple set of correlation parameters.

When we employ the Generalized DCC-MIDAS model, we obtain the parameters’ estimates reported in table 5. These estimates seem to confirm the need for a second MIDAS filter to be applied to the correlation between the manufacturing and the shops portfolios. Figure 10 shows that the long-run correlations filtered using this specification appear to be a little smoother when the 10-year bond is one of the assets compared to the results obtained under the previous specification. The low-frequency correlation of the two portfolios is instead a little noisier. Aside for these small differences, we take the results as confirming the need for multiple set of correlation parameters.

6 Concluding remarks

We introduced a class of DCC-MIDAS component models of dynamic correlations with a short- and long-run component specification. The key ingredients are a combination of the Engle (2002) DCC model, the Engle and Lee (1999) component GARCH model to replace the original DCC dynamics with a component specification and the Engle, Ghysels, and Sohn (2006) GARCH-MIDAS component specification that allows us to extract a long-run correlation component via mixed data sampling. We addressed the specification, estimation and interpretation of correlation models that distinguish short and long run components. We show that the changes in correlations are indeed very different. An empirical illustration shows the benefits of the component specification. Empirical specification tests are introduced and applied. They reveal the superior empirical fit, both in- and out-of-sample of
the new class of DCC-MIDAS correlation models. While we left the regularity conditions that guarantee standard asymptotic results for the two-step estimation of DCC-MIDAS as an open question for future research we did cover one important part of the regularity conditions dealing with the positive definiteness of the MIDAS-filtered long run correlation component.

References


Technical Appendix

A Positive semi-definiteness conditions

Proof of Proposition 1. Any matrix $C_t$ is positive semi-definite by construction. Therefore it must be the case that if $\forall j \in \{t - Lc, t\}$, $\exists C_j$ that is strictly positive definite, the resulting $\overline{R}_t (\omega^a_r)$ is positive definite.

Rewrite $\overline{R}_t (\omega^a_r, \omega^b_r)$ as

$$\overline{R}_t (\omega^a_r, \omega^b_r) = \left[ \begin{array}{cc} \overline{R}^{pp}_t (\omega^a_r) & \overline{R}^{pN}_t (\omega^a_r) \\ \overline{R}^{Np}_t (\omega^a_r)' & \overline{R}^{NN}_t (\omega^a_r) + \Lambda_t (\omega^a_r, \omega^b_r) \end{array} \right]$$

where $\Lambda_t (\omega^a_r, \omega^b_r)$ is an $N - p$ square matrix with ones on the main diagonal and off diagonal elements $\lambda^{ij}_t (\omega^a_r, \omega^b_r)$ defined as

$$\lambda^{ij}_t (\omega^a_r, \omega^b_r) = \sum_{l=1}^{Lc} \left[ \varphi_l (\omega^b_r) - \varphi_l (\omega^a_r) \right] C_{ij}^{t-l}$$

By assumption, the first $p$ minors of $\overline{R}_t (\omega^a_r, \omega^b_r)$ are positive. Any following minor can be written as

$$M^j_t = f^j \left( \lambda^{(p+1)}_t, \ldots, \lambda^{(j-1)j}_t \right) + \left| \overline{R}^{jj}_t (\omega^a_r) \right|, \ \forall j > p$$

where $f^j \left( \lambda^{(p+1)}_t, \ldots, \lambda^{(j-1)j}_t \right)$ is a continuous polynomial such that $f^j (0, \ldots, 0) = 0$, and $\left| \overline{R}^{jj}_t (\omega^a_r) \right|$ is the determinant of the first square matrix of size $j$ of $\overline{R}_t (\omega^a_r)$. By assumption, $\left| \overline{R}^{jj}_t (\omega^a_r) \right| > 0$.

By continuity of $f^j$ it must be the case that there exists an open neighborhood of $\omega^b_r = \omega^a_r$ in which $M^j_t > 0$, $\forall j$ and $\omega^b_r$:

$$S^j_t = \left\{ \omega^b_r : \omega^b_r \neq \omega^a_r \wedge M^j_t > 0 \right\}$$

Therefore $S_t = \bigcap_{j=p+1}^{N} S^j_t$ is non-empty and non-singleton.

Proof of Corollary 1. It follows from Proposition 1 that there exist parameters $\omega^a_r$ and $\omega^c_r$, with
\( \omega^a \neq \omega^c \) such that

\[
\mathcal{R}_t(\omega^a, \omega^c) = \begin{bmatrix} \mathcal{R}^{pp}_t(\omega^a) & \mathcal{R}^{pN}_t(\omega^c) \\ \mathcal{R}^{pN}_t(\omega^c)' & \mathcal{R}^{NN}_t(\omega^c) \end{bmatrix}
\]

is positive definite, that is

\[
\left| \mathcal{R}^{ij}_t(\omega^a, \omega^c) \right| > 0, \quad \forall j = \{1, \ldots, N\}
\]

As in the proof to proposition 1, define

\[
\mathcal{R}_t(\omega^a, \omega^b, \omega^c) = \begin{bmatrix} \mathcal{R}^{pp}_t(\omega^a) & \mathcal{R}^{pN}_t(\omega^c) \\ \mathcal{R}^{pN}_t(\omega^c)' & \mathcal{R}^{NN}_t(\omega^c) + \Lambda_t(\omega^c, \omega^b) \end{bmatrix}
\]

where \( \Lambda_t(\omega^c, \omega^b) \) is an \( N - p \) square matrix with ones on the main diagonal and off diagonal elements \( \lambda^{ij}_t(\omega^c, \omega^b) \) defined as

\[
\lambda^{ij}_t(\omega^c, \omega^b) = \sum_{l=1}^{L_c} \left[ \varphi_l(\omega^c) - \varphi_l(\omega^b) \right] C^{ij}_{l-1}
\]

The proof concludes by noting that any minor of size larger than \( p \) can be written as

\[
M^i_t = f^j(\lambda^{(p+1)}_t, \ldots, \lambda^{(j-1)}_t) + \left| \mathcal{R}^{ij}_t(\omega^a, \omega^c) \right|, \quad \forall j > p
\]

Proof of Corollary 3. The minors of \( \mathbf{a} \mathbf{a}' \odot \xi_{i-1} \xi_{i-1}' \) can be decomposed as:

\[
M^i_t = \left| (\xi_{i-1} \xi_{i-1}')^ii \right| \cdot a_1^{(i-n_1)I_{i>n_1}} \ldots a_J^{(i-n_{J-1})I_{i>n_{J-1}}}, \quad \forall i = 1, \ldots, n
\]

where \( I_{i>n_j} \) is an indicator function that takes the value of 1 if \( i > n_j \) and is equal to 0 otherwise. By assumption the matrix \( \xi_{i-1} \xi_{i-1}' \) is always positive definite and \( a_1, a_2, \ldots, a_J \) are positive. Hence \( M^i_t > 0, \forall i \). The same decomposition applies to each \( \varphi(\omega) \varphi(\omega)' \odot C_t, (\mathbf{u}' - \mathbf{a} \mathbf{a}' - \mathbf{b} \mathbf{b}')\mathcal{R}_t(\omega), \) and \( \mathbf{b} \mathbf{b}' \odot Q_{i-1}^{ij} \). Since \( Q_{i-1}^{ij} \) is sum of PSD matrices it will always be PSD.
Table 1

Mean Absolute Errors

<table>
<thead>
<tr>
<th></th>
<th>Step</th>
<th>Large Step</th>
<th>Dbl Step</th>
<th>Dbl Step Long</th>
<th>Ramp</th>
<th>Short Cycle</th>
<th>Long Cycle</th>
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<tbody>
<tr>
<td>DCC</td>
<td>0.072</td>
<td>0.113</td>
<td>0.115</td>
<td>0.118</td>
<td>0.159</td>
<td>0.120</td>
<td>0.100</td>
</tr>
<tr>
<td>Original</td>
<td>(.056, .091)</td>
<td>(.093, .142)</td>
<td>(.101, .132)</td>
<td>(.100, .136)</td>
<td>(.142, .173)</td>
<td>(.106, .134)</td>
<td>(.084, .122)</td>
</tr>
<tr>
<td>DCC</td>
<td>0.070</td>
<td>0.107</td>
<td>0.114</td>
<td>0.116</td>
<td>0.160</td>
<td>0.120</td>
<td>0.101</td>
</tr>
<tr>
<td>MIDAS</td>
<td>(.053, .097)</td>
<td>(.088, .133)</td>
<td>(.099, .133)</td>
<td>(.099, .133)</td>
<td>(.143, .174)</td>
<td>(.106, .134)</td>
<td>(.084, .122)</td>
</tr>
</tbody>
</table>

Notes - Each entry represents the average Mean Absolute Error (MAE) obtained by simulating correlations using the pattern reported in the corresponding column and estimating them through the model in the corresponding row. The numbers in parenthesis are the 95% confidence intervals.

Table 2

Energy Portfolio vs. 10 Year Bond

<table>
<thead>
<tr>
<th></th>
<th>μ</th>
<th>α</th>
<th>β</th>
<th>θ</th>
<th>ω</th>
<th>m</th>
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<td>Energy</td>
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<td>0.084</td>
<td>0.812</td>
<td>0.198</td>
<td>12.459</td>
<td>0.548</td>
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<td></td>
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<td>(.021)</td>
<td>(.006)</td>
<td>(.027)</td>
<td>(.000)</td>
<td>(.049)</td>
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<td>0.919</td>
<td>0.203</td>
<td>2.566</td>
<td>0.296</td>
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<tr>
<td></td>
<td>(.011)</td>
<td>(.000)</td>
<td>(.006)</td>
<td>(.018)</td>
<td>(.001)</td>
<td>(.075)</td>
</tr>
<tr>
<td>DCC-MIDAS</td>
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<td>0.979</td>
<td>1.774</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
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<td>(.003)</td>
<td>(.469)</td>
<td>–</td>
<td>–</td>
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</tr>
<tr>
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<td>0.981</td>
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<td>–</td>
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</tr>
<tr>
<td></td>
<td>(.000)</td>
<td>(.000)</td>
<td>–</td>
<td>–</td>
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<td>–</td>
</tr>
</tbody>
</table>

Notes - The top panel reports the estimates of the GARCH-MIDAS coefficients for the Energy portfolio and 10 year Bond. The bottom panel reports the estimates of the DCC-MIDAS and original DCC parameters. The number of MIDAS lags is 36 for the GARCH processes and 144 for the DCC process. The sample covers 1971-07-15 until 2006-06-30.
### Table 3

**Energy, Hi-Tech and 10 Year Bond**

<table>
<thead>
<tr>
<th></th>
<th>$\mu$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\theta$</th>
<th>$\omega$</th>
<th>$m$</th>
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</thead>
<tbody>
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<td>0.087</td>
<td>0.804</td>
<td>0.199</td>
<td>12.602</td>
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<td></td>
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<td>(0.000)</td>
<td>(0.016)</td>
<td>(0.065)</td>
<td>(0.000)</td>
<td>(0.123)</td>
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<tr>
<td>Hi-Tech</td>
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<td>0.087</td>
<td>0.837</td>
<td>0.186</td>
<td>9.997</td>
<td>0.726</td>
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<td>(0.000)</td>
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<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.332)</td>
</tr>
<tr>
<td>Bond</td>
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<td>0.915</td>
<td>0.204</td>
<td>3.090</td>
<td>0.284</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.000)</td>
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<table>
<thead>
<tr>
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<th>$b$</th>
<th>$\omega$</th>
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<tbody>
<tr>
<td>DCC-MIDAS</td>
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<td>1.683</td>
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<tr>
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<td>(0.004)</td>
<td>(0.000)</td>
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<td>0.981</td>
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<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
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</tr>
</tbody>
</table>

Notes - The top panel reports the estimates of the GARCH-MIDAS coefficients for the Energy portfolio, Hi-Tech portfolio and 10 year Bond. The bottom panel reports the estimates of the DCC-MIDAS and original DCC parameters. The number of MIDAS lags is 36 for the GARCH processes and 144 for the DCC process. The sample covers 1971-07-15 until 2006-06-30.
Table 4  
10 Year Bond, Manufacturing and Shops

<table>
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<tr>
<th></th>
<th>$\mu$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\theta$</th>
<th>$\omega$</th>
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</thead>
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<td>3.090</td>
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<td>(0.000)</td>
<td>(0.002)</td>
<td>(0.000)</td>
<td>(0.000)</td>
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<td>Manufacturing</td>
<td>0.072</td>
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<td>0.801</td>
<td>0.175</td>
<td>10.925</td>
<td>0.564</td>
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<tr>
<td></td>
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<td>(0.000)</td>
<td>(0.078)</td>
<td>(0.019)</td>
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<td>(0.000)</td>
<td>(0.003)</td>
<td>(0.001)</td>
<td>(0.013)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$a$</th>
<th>$b$</th>
<th>$\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCC-MIDAS</td>
<td>0.029</td>
<td>0.954</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>DCC</td>
<td>0.217</td>
<td>0.975</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

Notes - The top panel reports the estimates of the GARCH-MIDAS coefficients for the 10 year Bond, Manufacturing portfolio, and the Shops portfolio. The bottom panel reports the estimates of the DCC-MIDAS and of the original DCC parameters. The number of MIDAS lags is 36 for the GARCH processes and 144 for the DCC process. The sample covers 1971-07-15 until 2006-06-30.
### Table 5
10 Year Bond, Manufacturing, and Shops: multiple MIDAS and DCC

<table>
<thead>
<tr>
<th>Model</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
<th>( \omega_1 )</th>
<th>( \omega_2 )</th>
<th>Log-Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 DCC - 1 MIDAS</td>
<td>0.029</td>
<td>-</td>
<td>0.954</td>
<td>-</td>
<td>11.680</td>
<td>-</td>
<td>-9242.530</td>
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<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td>1 DCC - 2 MIDAS</td>
<td>0.030</td>
<td>-</td>
<td>0.950</td>
<td>-</td>
<td>7.585</td>
<td>26.300</td>
<td>-9237.999</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td>2 DCC - 1 MIDAS</td>
<td>0.025</td>
<td>0.031</td>
<td>0.958</td>
<td>0.958</td>
<td>8.039</td>
<td>-</td>
<td>-9238.499</td>
</tr>
<tr>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.013)</td>
<td>(0.004)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td>2 DCC - 2 MIDAS</td>
<td>0.025</td>
<td>0.036</td>
<td>0.958</td>
<td>0.942</td>
<td>7.451</td>
<td>26.023</td>
<td>-9235.319</td>
</tr>
<tr>
<td>(0.002)</td>
<td>(0.000)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.015)</td>
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</tr>
<tr>
<td>G-DCC-MIDAS</td>
<td>0.003</td>
<td>0.007</td>
<td>0.965</td>
<td>0.946</td>
<td>5.201</td>
<td>21.548</td>
<td>-9230.593</td>
</tr>
<tr>
<td>(2 DCC - 2 MIDAS)</td>
<td>(0.008)</td>
<td>(0.000)</td>
<td>(0.002)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
</tr>
</tbody>
</table>

**Bivariate Models**

<table>
<thead>
<tr>
<th>Model</th>
<th>( a )</th>
<th>( b )</th>
<th>( \omega )</th>
<th>Log-Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond vs Manufacturing</td>
<td>0.038</td>
<td>0.946</td>
<td>7.037</td>
<td>-9733.577</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bond vs Shops</td>
<td>0.038</td>
<td>0.936</td>
<td>7.608</td>
<td>-9818.094</td>
</tr>
<tr>
<td>(0.001)</td>
<td>(0.003)</td>
<td>(0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manufacturing vs Shops</td>
<td>0.031</td>
<td>0.952</td>
<td>24.587</td>
<td>-6939.103</td>
</tr>
<tr>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes - The top panel reports the estimates of the DCC-MIDAS coefficients for the 10 year Bond, Manufacturing, and Shops Portfolio for increasing number of DCC and MIDAS parameters. The second DCC and/or MIDAS set of parameters is applied to the correlation of Manufacturing vs Shops. The last two lines of the top panel report the estimation of the Generalized DCC-MIDAS. The bottom panel reports the estimates of the DCC-MIDAS model for the three bivariate systems obtained from all possible permutations of Bond, Manufacturing, and Shops. The number of MIDAS lags is 36 for the GARCH processes and 144 for the DCC process. The sample starts on 1971-07-15 and ends on 2006-06-30.
### Table 6
**Energy, Hi-Tech and 10 yrs Bond: Likelihood Ratio Tests**

<table>
<thead>
<tr>
<th></th>
<th>1 DCC</th>
<th>1 DCC</th>
<th>2 DCC</th>
<th>2 DCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 MIDAS</td>
<td>-</td>
<td>0.132</td>
<td>0.499</td>
<td>0.128</td>
</tr>
<tr>
<td>2 MIDAS</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.180</td>
</tr>
<tr>
<td>2 DCC</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.038</td>
</tr>
<tr>
<td>1 MIDAS</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes - Each entry represents the p-value for testing that the likelihood of the model on the column is significantly higher than the likelihood of the model on the corresponding row.

### Table 7
**10 yrs Bond, Manufacturing and Shops: Likelihood Ratio Tests**

<table>
<thead>
<tr>
<th></th>
<th>1 DCC</th>
<th>1 DCC</th>
<th>2 DCC</th>
<th>2 DCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 MIDAS</td>
<td>-</td>
<td>0.003</td>
<td>0.018</td>
<td>0.002</td>
</tr>
<tr>
<td>2 MIDAS</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.069</td>
</tr>
<tr>
<td>2 DCC</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.012</td>
</tr>
<tr>
<td>1 MIDAS</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes - Each entry represents the p-value for testing that the likelihood of the model on the column is significantly higher than the likelihood of the model on the corresponding row.
Fig. 1 - Simulated Correlations. The six subplots report the patterns of simulated correlations.
**Fig. 2** - Log-likelihoods of variances and correlations for increasing number of MIDAS lags. The left panel reports the log-likelihoods obtained by changing the number of GARCH-MIDAS lags for the energy portfolio and the 10 year bond variance and the correlation estimator, respectively. The right panel shows the log-likelihood of the correlation estimator for increasing DCC-MIDAS lags.
Fig. 3 - Long and short run volatilities and correlations for the energy portfolio and the 10 year bond. The pictures on the main diagonal refer to conditional variances of the energy portfolio and of the 10 year bond and the one on the off diagonal reports conditional correlations. In each panel the dark line refers to the long run and the light line represents the short run.
Fig. 4 - Differences in short run dynamics. The solid line represents the difference of short run correlations computed according to the DCC-MIDAS and the original DCC. The dashed line represents the excess long run correlation obtained as the difference between the DCC-MIDAS and the unconditional correlation.
Fig. 5 - Long and short run volatilities and correlations for the energy and hi-tech portfolios and the 10 year bond. The pictures on the main diagonal refer to conditional variances of energy and hi-tech portfolios and of 10 year bond and those on the off diagonal report conditional correlations among the same group of asset returns. In each panel the dark line refers to the long run and the light line represents the short run.
Fig. 6 - Differences in short run dynamics. In each subplot, the solid line represents the difference of short run correlations computed according to the DCC-MIDAS and the original DCC. The dashed line represents the excess long run correlation obtained as the difference between the DCC-MIDAS and the unconditional correlation. Starting from the top-left corner, the three subplots refer to the correlation of energy and hi-tech portfolios, energy portfolio and 10 year bond and hi-tech portfolio and 10 year bond.
Fig. 7 - Long and short run volatilities and correlations for the 10 year bond and Manufacturing and Shops portfolios. The pictures on the main diagonal refer to conditional variances of bond, manufacturing and shops and those on the off diagonal report conditional correlations among the same group of asset returns. In each panel the dark line refers to the long run and the light line represents the short run.
Fig. 8 - Differences in short run dynamics. In each subplot, the solid line represents the difference of short run correlations computed according to the DCC-MIDAS and the original DCC. The dashed line represents the excess long run correlation obtained as the difference between the DCC-MIDAS and the unconditional correlation. Starting from the top-left corner, the three subplots refer to the correlation of bond and manufacturing portfolio, bond and shops portfolio, and manufacturing and shops portfolios.
Fig. 9 - Long and short run volatilities and correlations for the 10 year bond and Manufacturing and Shops portfolios with 2 MIDAS filters. The second MIDAS filter is applied to the correlation between the manufacturing and the shops portfolios. The pictures on the main diagonal refer to conditional variances of bond, manufacturing and shops and those on the off diagonal report conditional correlations among the same group of asset returns. In each panel the dark line refers to the long run and the light line represents the short run.
Fig. 10 - Long and short run volatilities and correlations for the 10 year bond and Manufacturing and Shops portfolios using the Generalized DCC-MIDAS with 2 DCC set of parameters and 2 MIDAS filters. The second set of parameters is applied to the correlation between the manufacturing and the shops portfolios. The pictures on the main diagonal refer to conditional variances of bond, manufacturing and shops and those on the off diagonal report conditional correlations among the same group of asset returns. In each panel the dark line refers to the long run and the light line represents the short run.